1970-12

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Derin, Cetin

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Cetin Derin

United States
Naval Postgraduate School

THESIS

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Cetin Derin
Lieutenant (junior grade), Turkish Navy

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1970

ABSTRACT

An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotatory inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.

Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.

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ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. J. E. Brock for the invaluable aid and counsel which he has offered during the preparation of this thesis.

I. INTRODUCTION

A. GENERAL

In order to assure long time integrity of nuclear piping, the applicable codes, ANSI B31.7 - 1969 Code for Nuclear Power Piping and forthcoming edition of Section III of the ASME Boiler and Pressure Vessel Code, call for seismic analysis of such piping and require that the severity of the effects of seismically induced vibration be maintained below specified acceptable levels. Generally the problem of vibration analysis of the major piping systems is so complex that a major computational effort is required and this demands digital computer analysis using quite sophisticated programs. However, in the early stages of a design, when alternate layouts are being investigated, it is useful to be able to make some reasonable estimates of what a more elaborate analysis would be likely to reveal.

A recent document [3] concerned with assisting a designer to understand and comply with ANSI B31.7 contains material summarizing the vibration characteristics of several simple piping configurations. It suggests how the designer might employ this sort of information to assist him in estimating the vibration characteristics of more complicated configurations. This material should prove to be quite useful.

However, the scope of the configurations about which such information is available is quite limited. One of the

basic configurations with which a designer deals is a simple ell-shaped configuration. It was decided that it would be quite useful to determine and present in a concise form the significant results relating to the vibration of such a configuration. Specifically, it is assumed that the two ends of the configuration are perfectly fixed or clamped, that the pipe is of uniform properties along its entire length and is of rather small diameter compared to its length (hence the word "filament" in the title of this thesis) so that shear deflection and rotatory inertia could be neglected, and that the right angle junction of the two legs of the ell is small compared to the overall dimensions of the ell itself, so that it is not necessary to consider the properties of the bend or elbow that would exist in practice at the junction.

Such results are obtained and given herein. The basic parameter is the ratio of the length of the shorter leg to that of the longer leg; this parameter is designated by the Greek letter rho in what follows. A second important parameter, Greek beta in what follows, is required to relate the coupling that takes place between bending and torsion in the case of what are called out-of-plane vibrations.

This simple ell-shaped configuration has the great virtue of being such that the vibration problem is naturally divided into two disjoint sub-parts. One such part involves only displacements which lie in the plane of the ell; no torsion is involved in these cases. The other part involves

both bending and torsion, but the motion of any point on the axis of the filament is only perpendicular to the plane of the ell. This gives rise to the two terms used in what follows: "in-plane vibrations" and "out-of-plane vibrations."

B. ASSUMPTIONS AND LIMITATIONS

It is well to repeat here specifically the assumptions and limitations involved in this investigation.

1. The configuration is formed from originally straight uniform elastic members which meet at a right angle.
2. The angle at which the two members meet remains a right angle throughout the deformation of the configuration.
3. The ends of the configuration are assumed to be perfectly clamped.
4. The effect of shear deformation has been neglected. Only bending and torsional compliances are considered and deformations are assumed to be "small."
5. Rotatory inertia has been neglected.
6. Poisson's ratio has been taken to be 0.3.

Additionally, there are limitations implicit in the fact that values of the roots of the equations which are later solved are obtained only to an accuracy of one part in one million parts and in the fact that the computational programs used a mantissa having 56 bits. These limitations resulted in being able to obtain only about the lowest

thirteen frequencies for a particular configuration. Also, only a few values of beta were investigated and there are some difficulties in the use of linear interpolation for other values of beta.

C. NOTATION

A, B, C, D, F, H	Constants in equation (2-5) and (2-14)
E	Modulus of elasticity
G	Shear modulus of elasticity
g_1, g_2, g_3, g_4	Functions defined in equation (2-6)
I	Moment of inertia of cross section
J	Polar moment of inertia
L	Length of pipe
M	Bending moment
P	Defined in equation (2-4)
t	Time
T	Torque
V	Shear force
x	Distance from fixed end
y	Lateral deflection
Y	Lateral deflection mode shape
Y'	Slope
α	Defined in equation (2-13)
β	Defined in equation (2-29)
γ	Phase angle in equation (2-11)
θ	Angle of twist
ϕ	Angle of twist mode shape

ψ	Constant in equation (2-2)
μ	Mass per unit length
ν	Mass moment of inertia per unit length
ρ	Length ratio
ω	Circular frequency

II. THEORETICAL DEVELOPMENTS

A. BENDING ANALYSIS

Neglecting the effects of shear deflection and rotatory inertia the equation governing lateral vibration of a uniform beam is

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = 0 \tag{2-1}$$

where μ is mass per unit length, EI is flexural rigidity, y is lateral deflection and t is time. Assuming harmonic oscillation with circular frequency ω, the solution of equation (2-1) can be written in the following form:

$$y(x,t) = Y(x) \cos(\omega t + \psi) \tag{2-2}$$

where ψ is a constant and Y is a function of x alone. Substitution of equation (2-2) into equation (2-1) yields

$$\frac{d^4 Y(x)}{dx^4} - P^4 Y(x) = 0 \tag{2-3}$$

where

$$P^4 = \frac{\mu \omega^2}{EI} \tag{2-4}$$

The general solution of equation (2-3) may be written as

$$Y(x) = Ag_1 + Bg_2 + Cg_3 + Dg_4 \tag{2-5}$$

where

$$g_1 = \tfrac{1}{2}[\mathrm{Cosh}(Px) + \mathrm{Cos}(Px)] \qquad (a)$$

$$g_2 = \tfrac{1}{2}[\mathrm{Sinh}(Px) - \mathrm{Sin}(Px)] \qquad (b)$$

$$g_3 = \tfrac{1}{2}[\mathrm{Cosh}(Px) - \mathrm{Cos}(Px)] \qquad (c) \qquad (2\text{-}6)$$

$$g_4 = \tfrac{1}{2}[\mathrm{Sinh}(Px) + \mathrm{Sin}(Px)] \qquad (d)$$

At any location x, deflection Y_x, slope Y'_x, bending moment M_x, and shear force V_x can be expressed as

$$Y_x = Ag_1 + Bg_2 + Cg_3 + Dg_4 \qquad (a)$$

$$Y'_x = P(Ag_2 + Bg_3 + Cg_4 + Dg_1) \qquad (b)$$

$$M_x = EIP^2(Ag_3 + Bg_4 + Cg_1 + Dg_2) \qquad (c) \qquad (2\text{-}7)$$

$$V_x = EIP^3(Ag_4 + Bg_1 + Cg_2 + Dg_3) \qquad (d)$$

At $x = 0$,

$$Y_0 = A \quad ; \quad Y'_0 = DP \qquad (a,b)$$

$$M_0 = C(EIP^2) \quad ; \quad V_0 = B(EIP^3) \qquad (c,d) \qquad (2\text{-}8)$$

Expressing the constants A, B, C and D in terms of Y_0, Y'_0, M_0 and V_0 and substituting into equation (2-7) yields:

$$\begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_x = \begin{bmatrix} g_1 & g_4/P & g_3/EIP^2 & g_2/EIP^3 \\ Pg_2 & g_1 & g_4/EIP & g_3/EIP^2 \\ EIP^2g_3 & EIPg_2 & g_1 & g_4/P \\ EIP^3g_4 & EIP^2g_3 & Pg_2 & g_1 \end{bmatrix} \begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_0 \qquad (2\text{-}9)$$

The above equation expresses the deflection, slope, bending moment, and shear force at any location x in terms of those at x = 0.

B. TORSION ANALYSIS

The equation of motion of a uniform cylinder undergoing torsional vibration is,

$$\frac{\partial^2\theta}{\partial x^2} - \frac{\nu}{JG}\frac{\partial^2\theta}{\partial t^2} = 0 \tag{2-10}$$

where ν is mass moment of inertia per unit length, J is polar moment of inertia of elastic resistance, and θ is angle of twist. For a sinusoidal solution at circular frequency ω,

$$\theta(x,t) = \phi(x)\ \mathrm{Cos}(\omega t + \gamma) \tag{2-11}$$

where γ is a constant and ϕ is a function of x alone. Substitution in equation (2-10) gives

$$\frac{d^2\phi(x)}{dx^2} + \alpha^2\phi(x) = 0 \tag{2-12}$$

where

$$\alpha^2 = \frac{\nu\omega^2}{JG} \tag{2-13}$$

The general solution of equation (2-12) is

$$\phi(x) = F\ \mathrm{Cos}(\alpha x) + H\ \mathrm{Sin}(\alpha x) \tag{2-14}$$

and torque T is

$$T = JG\frac{d\phi(x)}{dx} = JG\alpha[-F\ \mathrm{Sin}(\alpha x) + H\ \mathrm{Cos}(\alpha x)] \tag{2-15}$$

At $x = 0$

$$\phi_0 = F \quad ; \quad T_0 = H(JG\alpha) \qquad \text{(2-16a,b)}$$

Expressing constants F and H in terms of ϕ_0 and T_0 and substituting into equations (2-14) and (2-15), the angle of twist ϕ_x and torque T_x at any location x, can be expressed in terms of T_0 and ϕ_0 as follows:

$$\begin{bmatrix} \phi \\ T \end{bmatrix}_x = \begin{bmatrix} \text{Cos}(\alpha x) & \text{Sin}(\alpha x)/JG\alpha \\ -JG\alpha\ \text{Sin}(\alpha x) & \text{Cos}(\alpha x) \end{bmatrix} \begin{bmatrix} \phi \\ T \end{bmatrix}_0 \qquad \text{(2-17)}$$

C. IN-PLANE AND OUT-OF-PLANE VIBRATION ANALYSIS OF AN ELL-SHAPED UNIFORM FILAMENT WITH CLAMPED ENDS

1. In-plane Vibrations

In Figure 2-1, an ell-shaped configuration and the corresponding free body diagrams are shown.

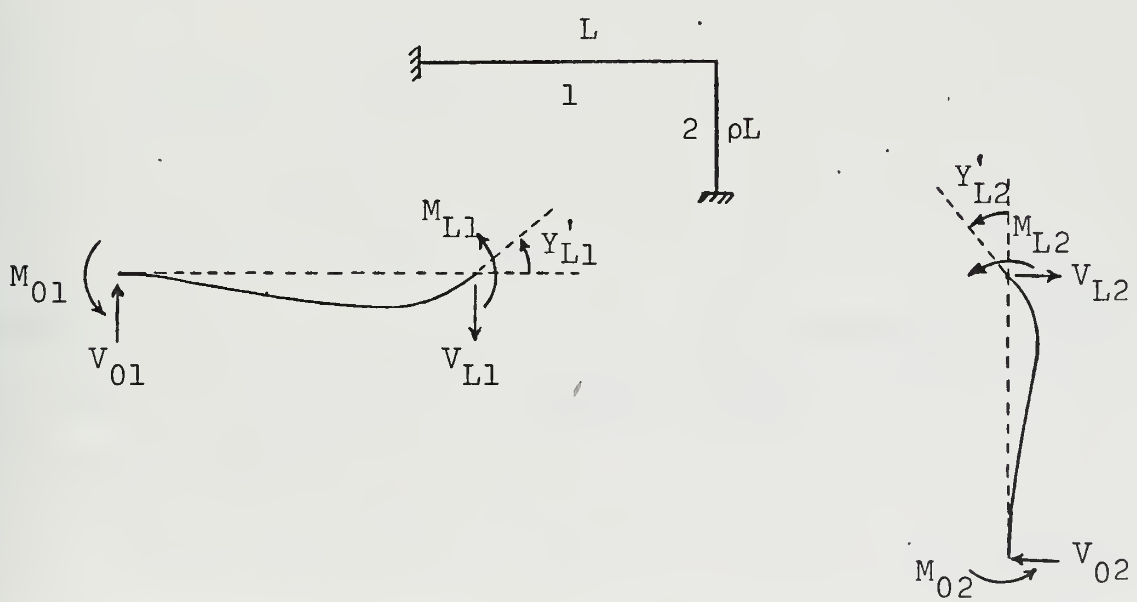


Figure 2-1. Ell-shaped Configuration and Free Body Diagrams for In-plane Vibrations.

The ratio, ρ, of the length of the shorter leg to that of the longer is a basic geometrical parameter in the following analysis.

At the clamped ends deflections Y_{01}, Y_{02} and slopes Y'_{01}, Y'_{02} are equal to zero. Also at the junction of the two portions, deflections Y_{L1} and Y_{L2} are equal to zero. Application of equation (2-9) to the two portions yields,

$$\left.\begin{aligned}
&\frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} = 0 && \text{(a)}\\
&\frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} = Y'_{L1} && \text{(b)}\\
&g_1 M_{01} + \frac{g_4}{P} V_{01} = M_{L1} && \text{(c)}\\
&\frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} = 0 && \text{(d)}\\
&\frac{\bar{g}_4}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^2} V_{02} = Y'_{L2} && \text{(e)}\\
&\bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} = M_{L2} && \text{(f)}
\end{aligned}\right\} \quad (2\text{-}18)$$

where g_i indicates evaluation of $g_i(x)$ for argument (PL) and $\bar{g}_i$ indicates evaluation for argument (ρPL).

At the junction,

$$Y'_{L1} = Y'_{L2} \quad ; \quad M_{L1} + M_{L2} = 0 \qquad (2\text{-}19a,b)$$

Combining equations (2-19) with equations (2-18), the following set of equations is obtained.

$$\begin{bmatrix} g_3/EIP^2 & 0 & g_2/EIP^3 & 0 \\ 0 & \bar{g}_3/EIP^2 & 0 & \bar{g}_2/EIP^3 \\ g_4/EIP & -\bar{g}_4/EIP & g_3/EIP^2 & -g_3/EIP^2 \\ g_1 & \bar{g}_1 & g_4/P & \bar{g}_4/P \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \qquad (2\text{-}20)$$

For a nontrivial solution, the determinant, D, of the above matrix must vanish. Elementary manipulations on D lead to the following form.

$$\begin{vmatrix} g_3 & 0 & g_2 & 0 \\ 0 & \bar{g}_3 & 0 & \bar{g}_2 \\ g_4 & -\bar{g}_4 & g_3 & -\bar{g}_3 \\ g_1 & \bar{g}_1 & g_4 & \bar{g}_4 \end{vmatrix} = 0 \qquad (2\text{-}21)$$

The terms of equation (2-21) are functions of ρ. For any value of ρ, the zeros $(PL)_i$ of the determinant can be found. The corresponding frequencies are evaluated by using equation (2-4) as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L^4}} \qquad (2\text{-}22)$$

Solution of equation (2-21) will be explained in Section III-A.

2. Out-of-plane Vibrations

Out-of-plane vibrations can be analyzed similarly. However in this case the effect of torsion must be taken into account.

In Figure 2-2, free body diagrams of an ell-shaped configuration for out-of-plane vibrations are shown.

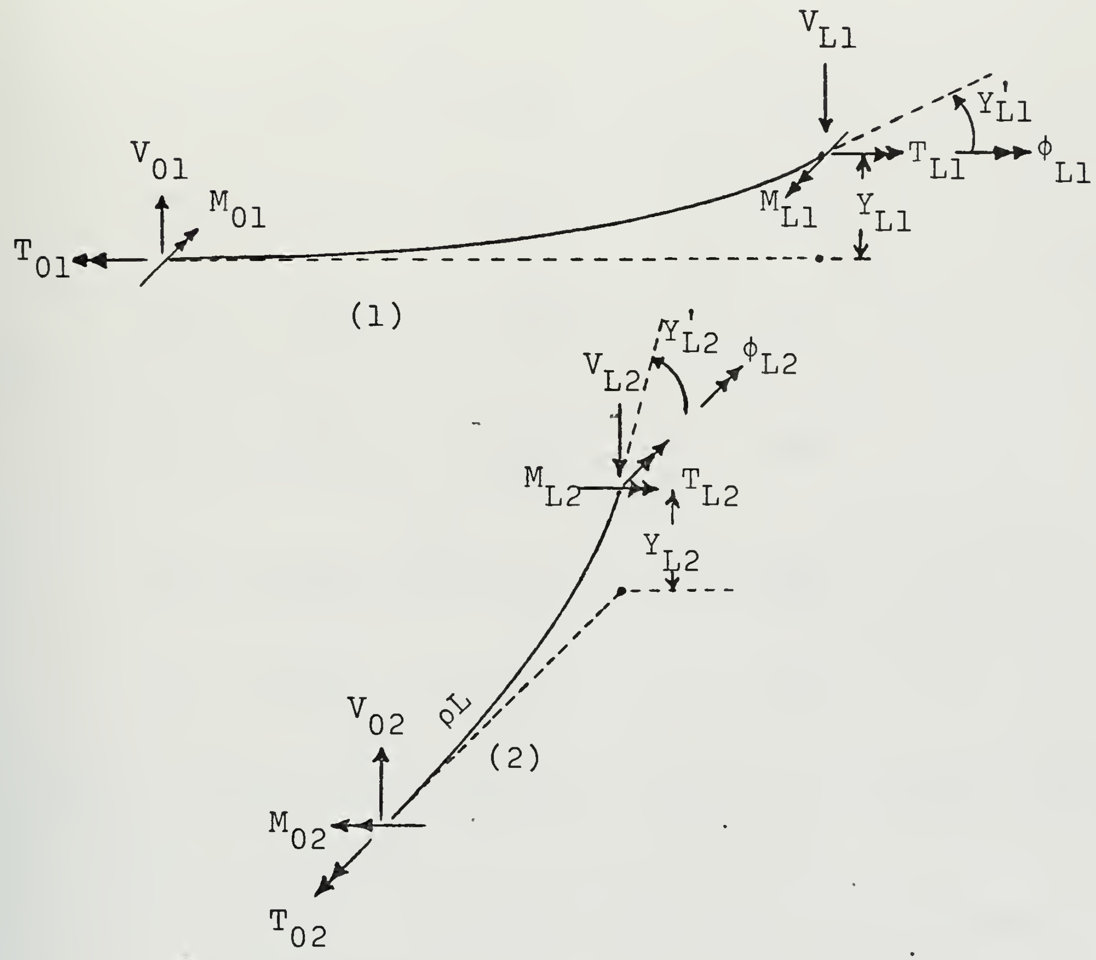


Figure 2-2. Free Body Diagrams of an Ell-shaped Configuration for Out-of-plane Vibrations.

At the clamped ends deflections Y_{01}, Y_{02} and slopes Y'_{01}, Y'_{02} and angles of twist ϕ_{01}, ϕ_{02} are equal to zero. Application of equation (2-9) to the two portions yields

$$Y_{L1} = \frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} \quad \text{(a)}$$

$$Y'_{L1} = \frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} \quad \text{(b)}$$

$$M_{L1} = g_1 M_{01} + \frac{g_4}{P} V_{01} \quad \text{(c)}$$

$$V_{L1} = P g_2 M_{01} + g_1 V_{01} \quad \text{(d)}$$

$$Y_{L2} = \frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} \quad \text{(e)} \qquad \text{(2-23)}$$

$$Y'_{L2} = \frac{\bar{g}_4}{EIP} M_{02} + \frac{\bar{g}_3}{EIP^2} V_{02} \quad \text{(f)}$$

$$M_{L2} = \bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} \quad \text{(g)}$$

$$V_{L2} = P\bar{g}_2 M_{02} + \bar{g}_1 V_{02} \quad \text{(h)}$$

where g_i indicates evaluation of $g_i(x)$ for argument (PL) and $\bar{g}_i$ indicates evaluation for argument (ρPL).

For the torsion, equation (2-17) applies and gives,

$$\phi_{L1} = \frac{\text{Sin}(\alpha L)}{JG\alpha} T_{01} \quad \text{(a)}$$

$$T_{L1} = \text{Cos}(\alpha L)\, T_{01} \quad \text{(b)} \qquad \text{(2-24)}$$

$$\phi_{L2} = \frac{\text{Sin}(\alpha\rho L)}{JG\alpha} T_{02} \quad \text{(c)}$$

$$T_{L2} = \text{Cos}(\alpha\rho L) T_{02} \quad \text{(d)}$$

At the junction:

$$M_{L1} = T_{L2} \quad ; \quad T_{L1} = -M_{L2} \qquad (a,b)$$

$$Y'_{L1} = -\phi_{L2} \quad ; \quad \phi_{L1} = Y'_{L2} \qquad (c,d) \qquad (2\text{-}25)$$

$$V_{L1} = -V_{L2} \quad ; \quad Y_{L1} = Y_{L2} \qquad (e,f)$$

Specializing to the case of circular cross section and assuming Poisson's ratio is 0.3, then

$$\frac{1}{JG\alpha} = \frac{1.3}{EI\alpha} \qquad (2\text{-}26)$$

Combination of equations (2-23), (2-24), (2-25) and (2-26) gives the following set of equations.

$$\begin{bmatrix} g_3P & -\bar{g}_3P & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4P & -\frac{1.3P^2}{\alpha}\mathrm{Sin}(\alpha L) & 0 & 0 & \bar{g}_3 \\ g_4P & 0 & 0 & \frac{1.3P^2}{\alpha}\mathrm{Sin}(\alpha\rho L) & g_3 & 0 \\ g_2P & \bar{g}_2P & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & -\mathrm{Cos}(\alpha\rho L) & \frac{g_4}{P} & 0 \\ 0 & \bar{g}_1 & \mathrm{Cos}(\alpha L) & 0 & 0 & \frac{\bar{g}_4}{P} \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ T_{01} \\ T_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \qquad (2\text{-}27)$$

For a nontrivial solution, the determinant D of this matrix must vanish. From equations (2-4), (2-13) and (2-26) it is found that

$$(\alpha L) = \beta(PL)^2 \qquad (2\text{-}28)$$

where

$$\beta = \sqrt{\frac{1.3\nu}{\mu L^2}} \tag{2-29}$$

is a dimensionless parameter. Substituting equation (2-28) into equation (2-27) and performing elementary manipulations on D give the following equation

$$\begin{vmatrix} g_3 & -\bar{g}_3 & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4 & -\frac{1.3}{\beta}\mathrm{Sin}[\beta(PL)^2] & 0 & 0 & g_3 \\ g_4 & 0 & 0 & \frac{1.3}{\beta}\mathrm{Sin}[\rho\beta(PL)^2] & g_3 & 0 \\ g_2 & \bar{g}_2 & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & (PL)\mathrm{Cos}[\beta\rho(PL)^2] & g_4 & 0 \\ 0 & \bar{g}_1 & (PL)\mathrm{Cos}[\beta(PL)^2] & 0 & 0 & \bar{g}_4 \end{vmatrix} = 0 \tag{2-30}$$

The terms of equation (2-30) are functions of ρ and β. For given values of ρ and β, the zeros $(PL)_i$ of determinant D can be found. The corresponding frequencies are evaluated by using equation (2-22).

Solution of equation (2-30) will be explained in Section III-A.

III. PRESENTATION OF RESULTS

A. METHODS OF SOLUTION

The equations (2-21) and (2-30), developed in Chapter II, can be solved either by algebraic manipulations or by using direct computer solution.

In a first approach to a solution, algebraic manipulations were performed to reduce the order of determinants. Then transcendental equations were obtained and two computer programs, one for the in-plane solution and one for the out-of-plane solution, were developed to get the roots of these equations.

In the computer programs the transcendental equations were used as function subprograms and roots located in an interval. Then the Newton-Raphson method was used to obtain good approximate values of the roots.

The procedure explained above led to some difficulties. During the solution of the equation which was derived from equation (2-30), the Newton-Raphson method failed to give an expected root because the slope of the transcendental function reached very high values and this caused a jump to adjacent roots. In one case, the number of iterations required exceeded three hundred with an allowable error of 10^{-8} (for the quantity PL) and increasing the allowable error to 10^{-6} did not make much difference. Finally, comparison of results with the classical solution in the case ρ is

equal to zero showed that some of the roots were extraneous.

The difficulties mentioned in the preceding paragraph were eliminated by making direct computer evaluation of the determinants (2-21) and (2-30). For this purpose subroutine BRODE was developed and each element of the determinants was stored in this subprogram. In addition to subroutine BRODE, library subroutine DTERM which was developed by Jean Bow in 1967 was used to evaluate the determinants. This modified procedure resulted in reduction of computer time required and in the elimination of extraneous roots.

Difficulties with occasional failure of the Newton-Raphson method were reduced by using the method of successive bisectioning. In this method the number of iterations required for one root did not exceed fifty with an allowable error of 10^{-6}. This procedure also resulted in conservation of computer time.

Detailed instructions for use of the computer programs are given in Appendix B.

B. GRAPHS

Numerical results given to a large number of significant figures were obtained primarily for the purpose of investigating the accuracy and integrity of the solutions. Chapter IV will give a discussion of these investigations.

However, so as to present the results in the most convenient form for practical use, graphical outputs

were also obtained by use of the library subroutine DRAW.

One graph was obtained for the in-plane solution. For the out-of-plane solution five graphs were obtained corresponding to five different values of β. In each case, the ordinates are values of (PL) and abscissas are values of ρ. On these graphs the first (lowest) curve is for the first mode, the second curve for the second mode and so on. Graphs are given in Figures 3-1 through 3-6.

C. SAMPLE PROBLEM

A sample problem will be solved to show how to use the graphs given in the preceding article.

Properties of ell-shaped configuration:

L_1	:	16' - 6"
L_2	:	10' - 3½"
Outside diameter of pipe	:	6.625"
Inside diameter of pipe	:	6.065"
Wall thickness of pipe	:	0.280"
Weight of pipe	:	18.98 lb/ft
Thickness of insulation	:	1 1/8"
Weight of insulation	:	2.90 lb/ft
Weight of contents	:	12.51 lb/ft
Modulus of elasticity E	:	27,400,000 lb/in^2
G	:	10,600,000 lb/in^2

For the use of graphs, values of ρ and β must be known. For this problem,

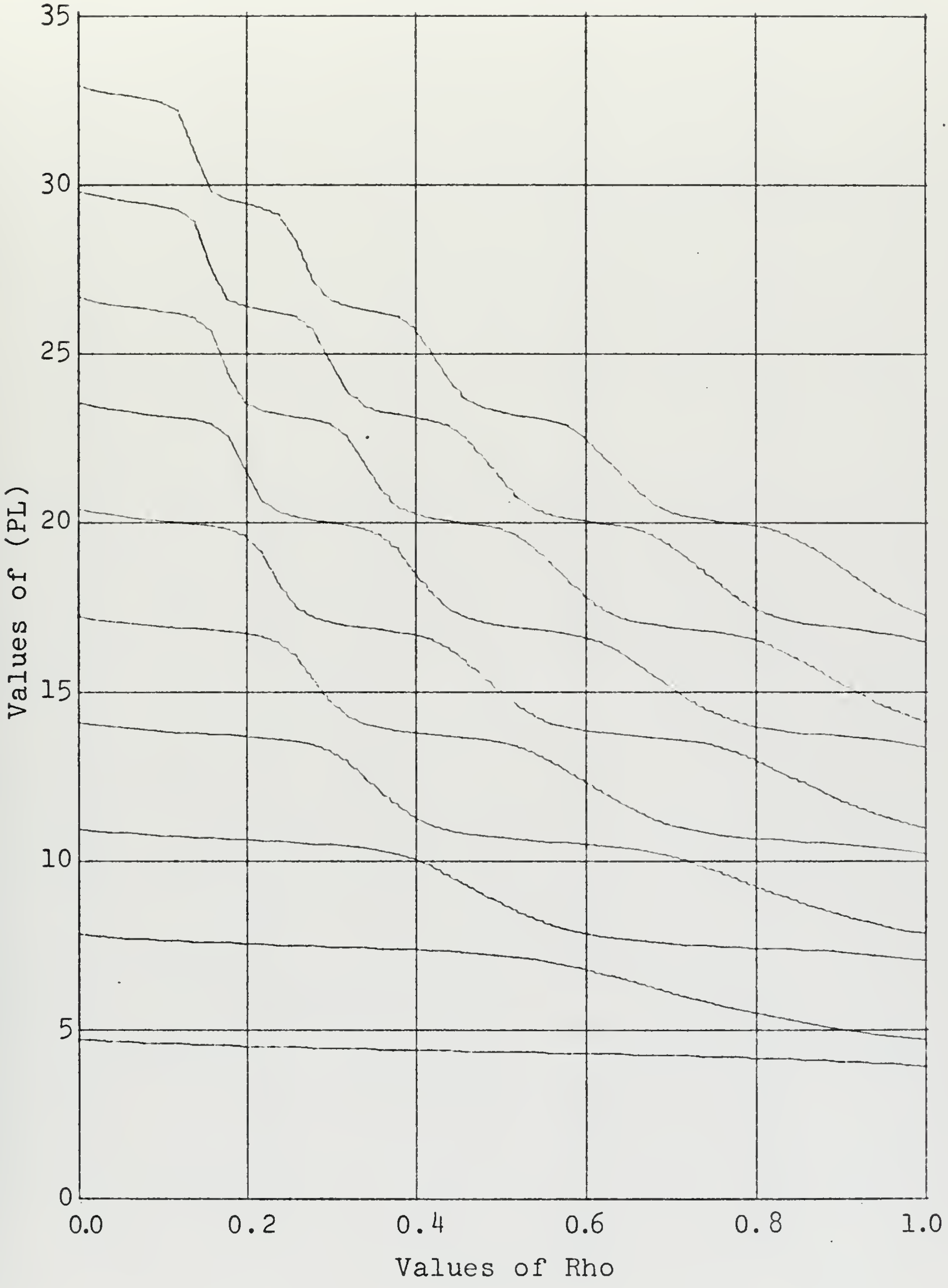


Figure 3-1. In-plane Vibrations.

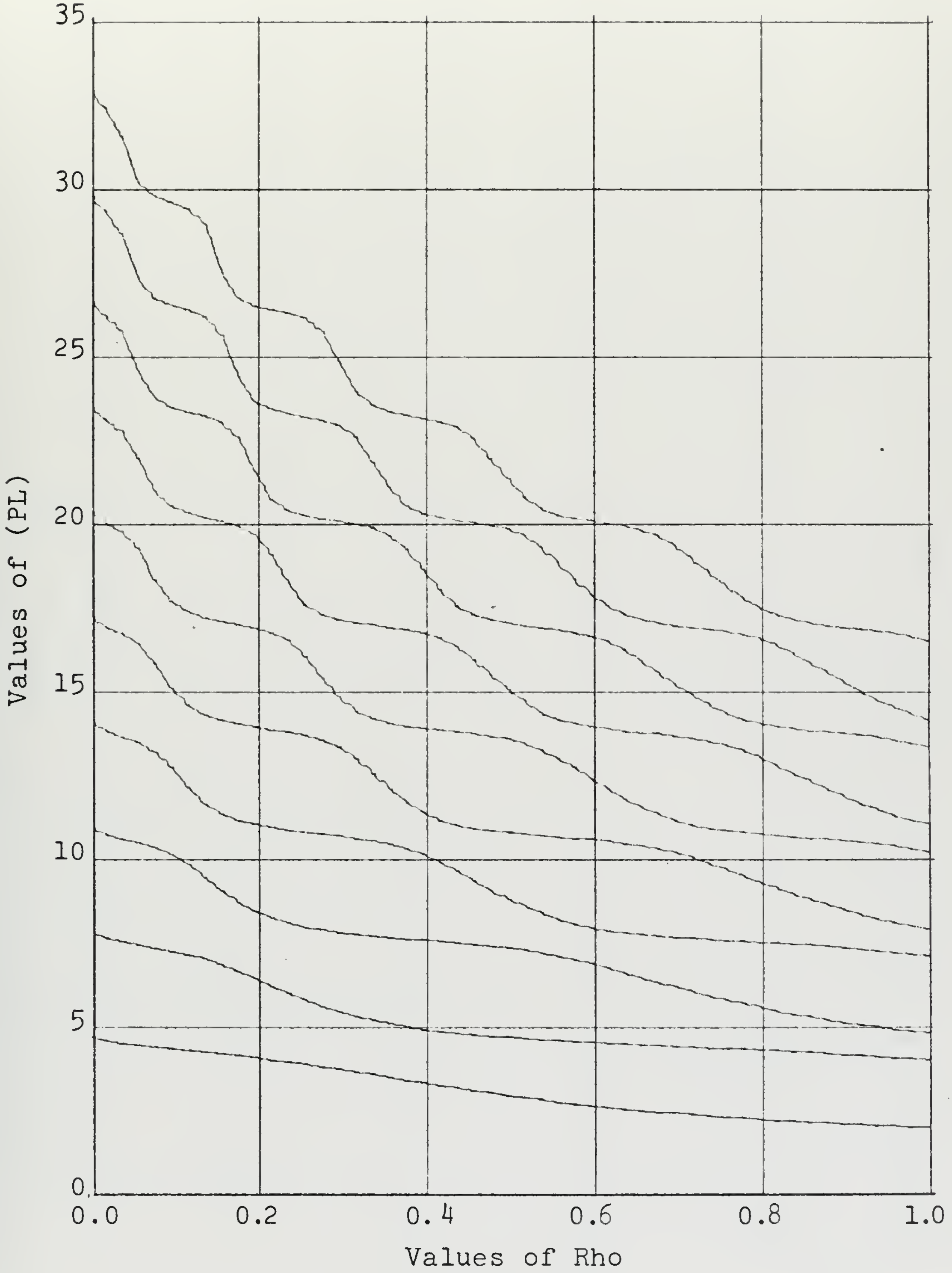


Figure 3-2. Out-of-plane Vibrations, Beta = 0.00001.

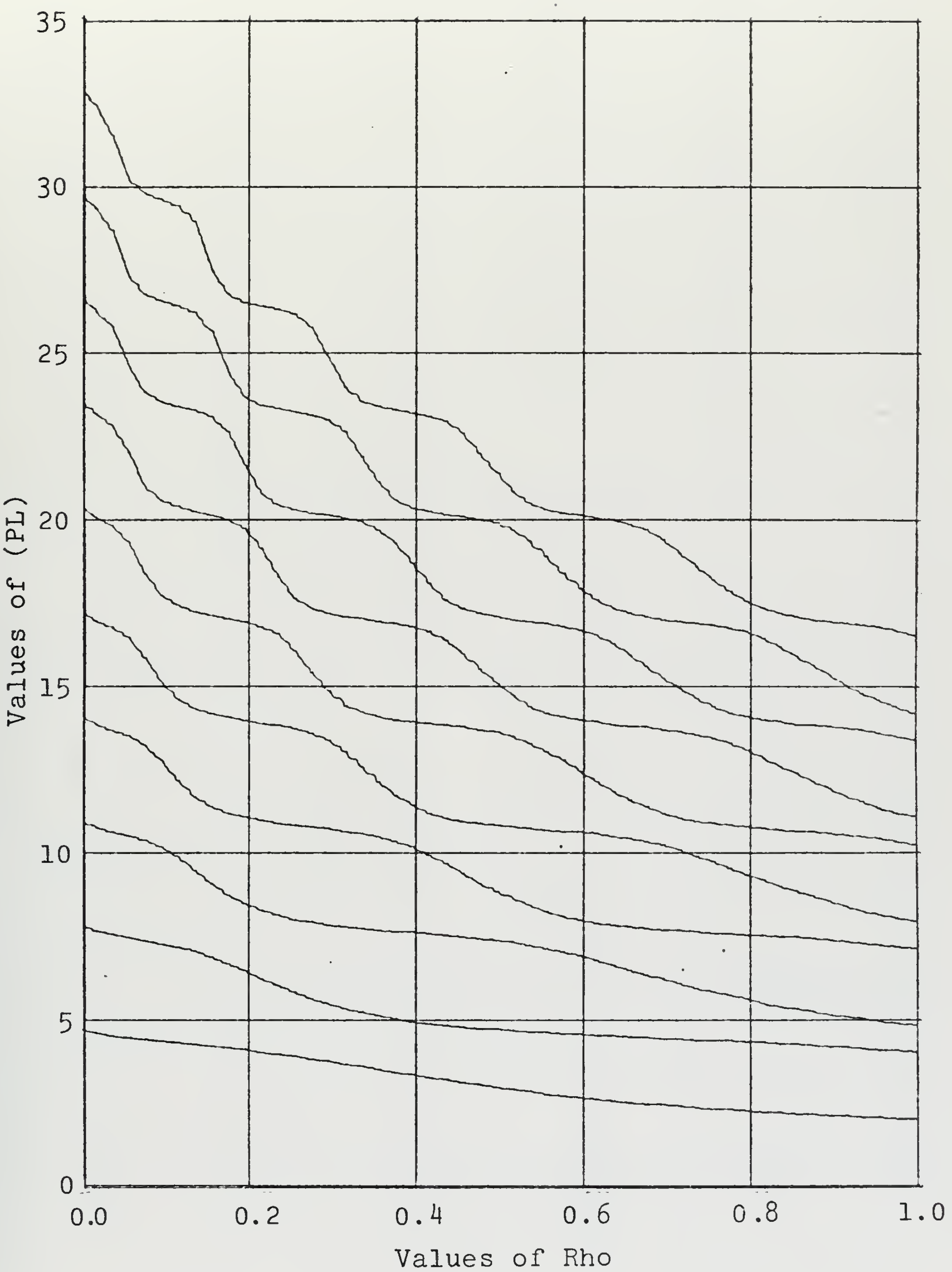


Figure 3-3. Out-of-plane Vibrations, β = 0.001.

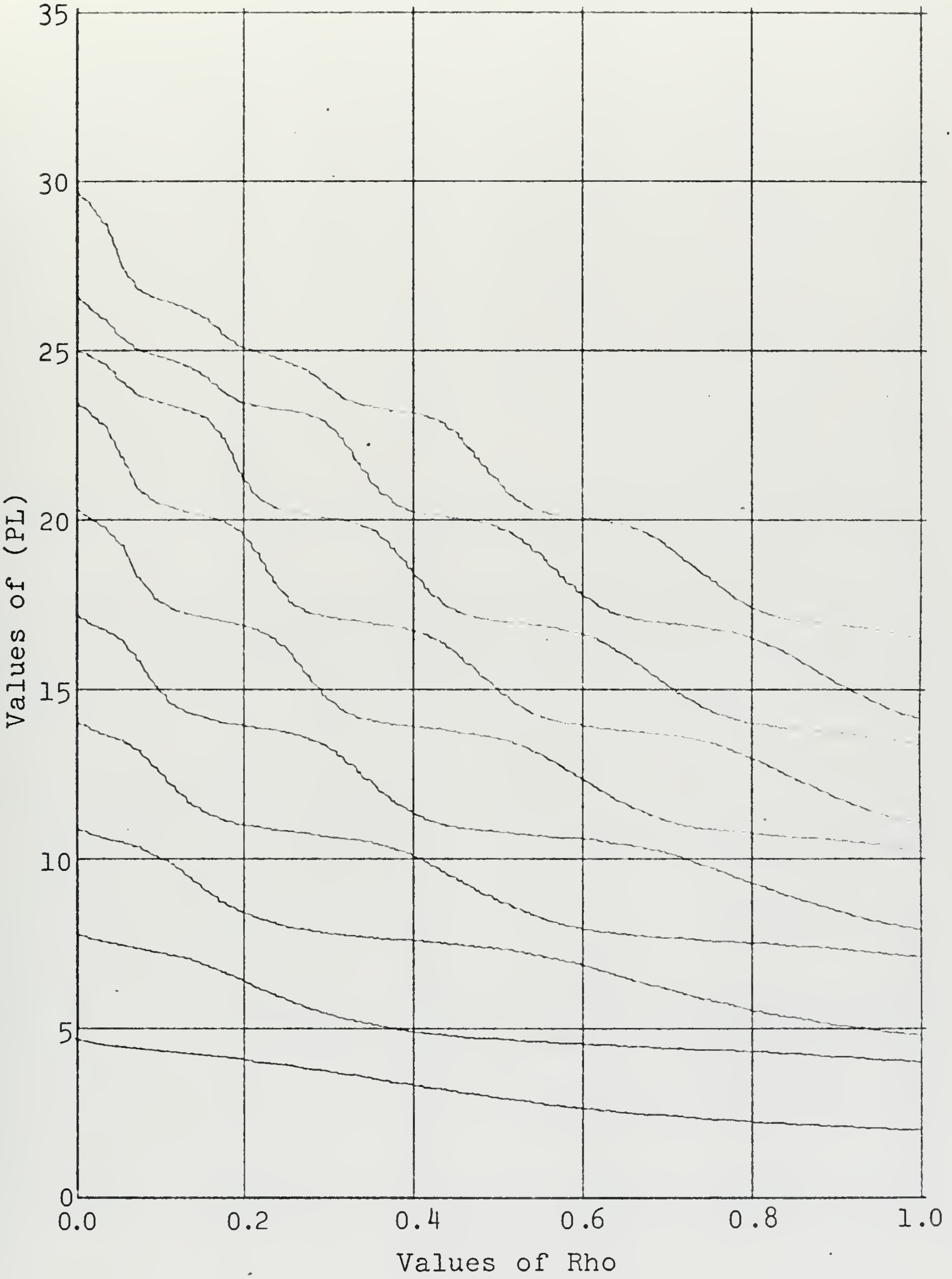


Figure 3-4. Out-of-plane Vibrations, β = 0.005.

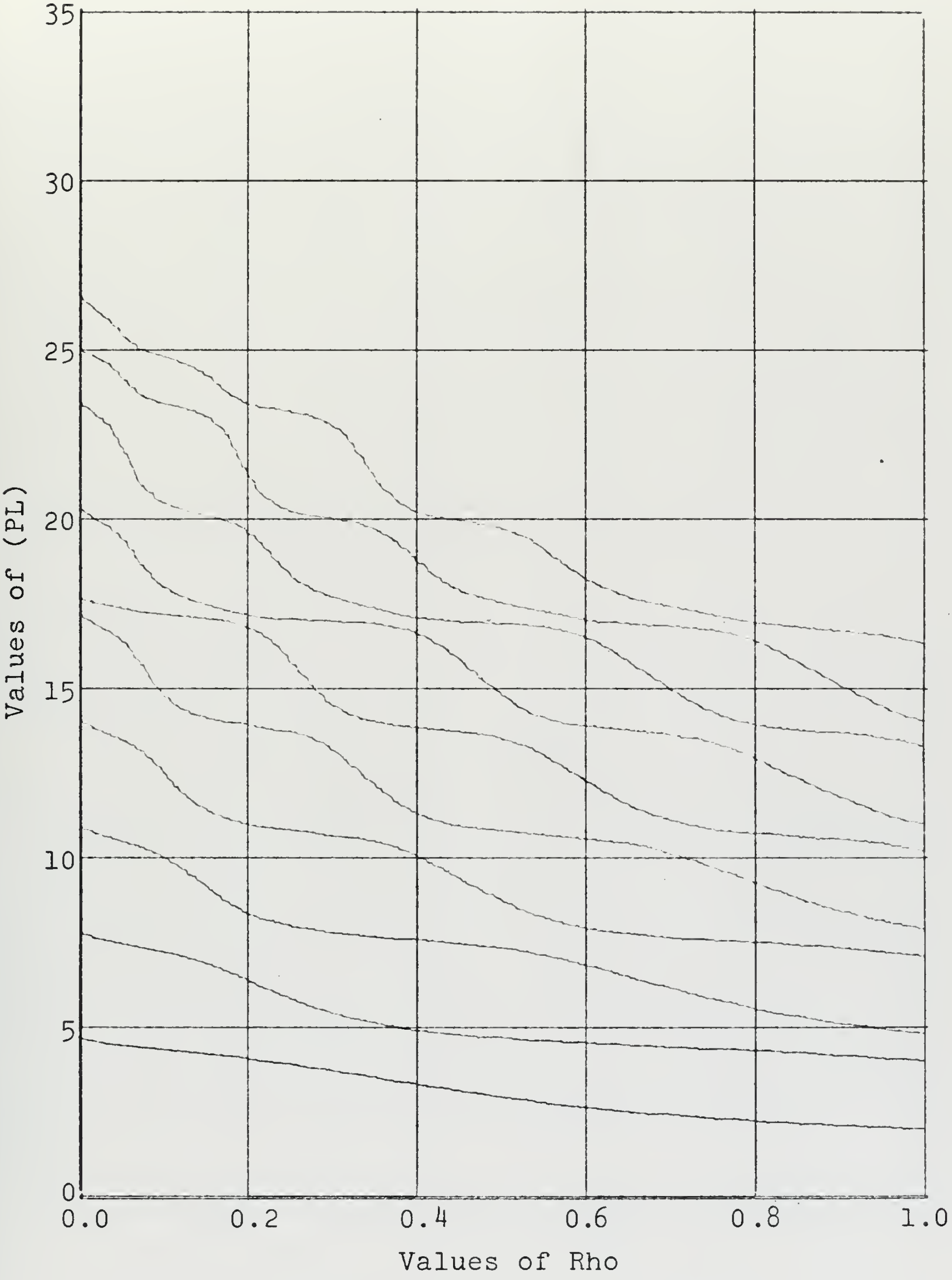


Figure 3-5. Out-of-plane Vibrations, β = 0.01.

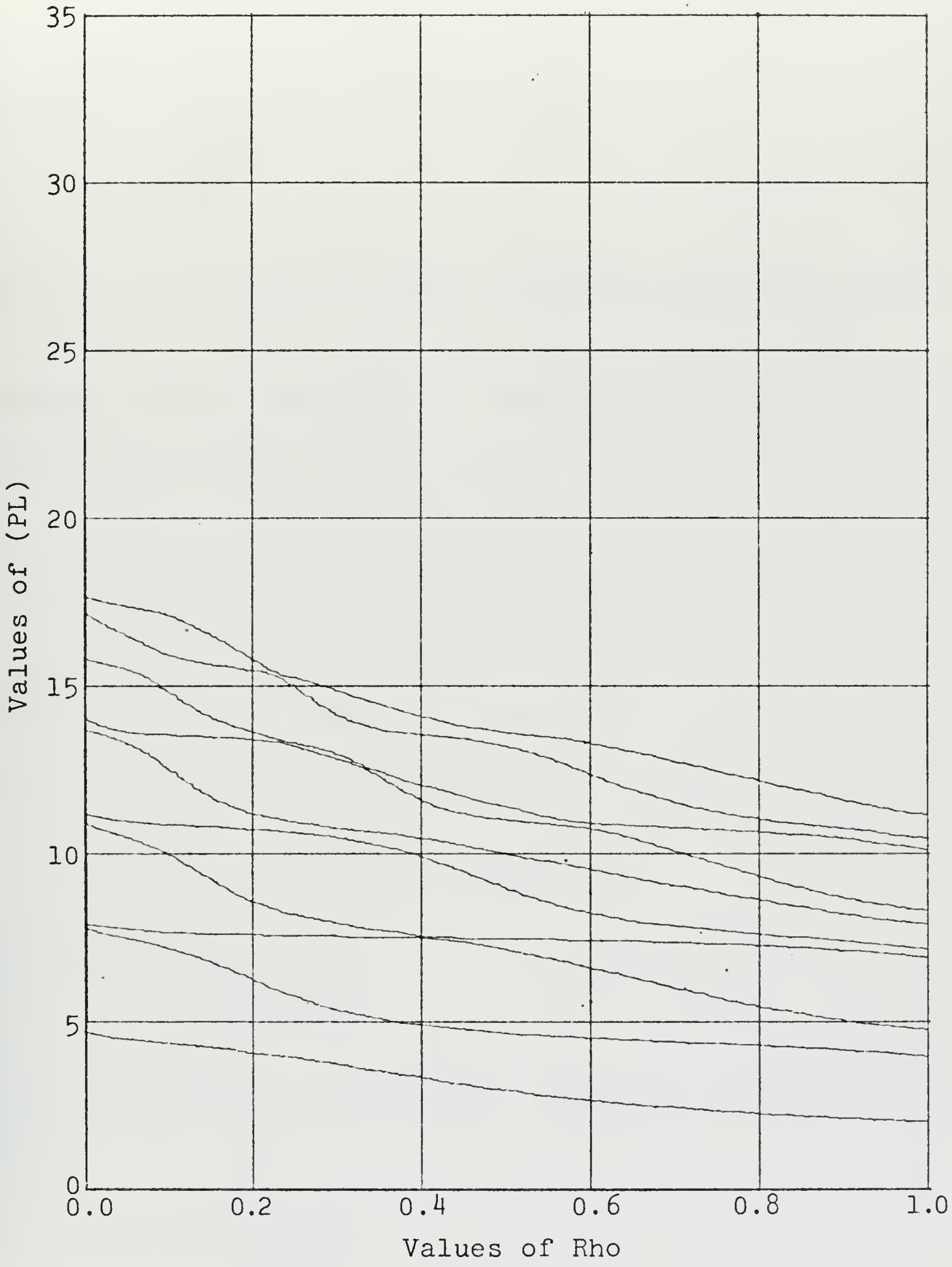


Figure 3-6. Out-of-plane Vibrations, β = 0.05.

$$\rho = \frac{L_2}{L_1} = 0.62373737.....$$

and β can be found from equation (2-29).

$$\beta = \sqrt{\frac{1.3\nu}{\mu L_1^2}} \tag{3-1}$$

where ν is the total mass moment of inertia per unit length for the torsion and μ is the total mass per unit length undergoing lateral vibrations. Assuming that the contents (water) do not rotate, then

$$\nu = \nu_{pipe} + \nu_{insulation}$$

$$\mu_{total} = \mu_{pipe} + \mu_{insulation} + \mu_{contents}$$

numerical values of ν_{total} and μ_{total} are

$$\nu_{total} = 0.0509089405 \text{ lb-sec}^2$$

$$\mu_{total} = 1.068875490 \text{ lb-sec}^2\text{-in}^{-2}$$

and from equation (3-1)

$$\beta = 0.0150806896$$

For evaluation of frequencies, equation (2-22) should be used as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L_1^4}}$$

then

$$\sqrt{\frac{EI}{\mu L_1^4}} = 8.221317961 \text{ sec}^{-1}$$

To solve the problem, values of $(PL)_i$ can be found from the graphs. But no graph is available for the value of β found above. Therefore interpolation is required between the values taken from Figure 3-5 and from Figure 3-6. Values of $(PL)_i$ are given in the table below where the exact solution was obtained by a computer solution using the specified value for β.

MODE NO.	EXACT SOLUTION	SOLUTION OBTAINED BY LINEAR INTERPOLATION
1	2.624579	2.624302
2	4.533513	4.521991
3	6.729044	6.706905
4	7.862686	7.817582
5	10.518163	10.230722
6	11.908733	11.698410
7	13.730277	13.442683
8	14.259158	15.619057
9	16.376500	16.379861
10	17.185014	17.403866

As seen from the table, for the lower modes, linear interpolation gives good results. However there is greater error in using linear interpolation for the higher frequencies. One can make what appears to be the reasonable conjecture that if one were to plot curves with (PL) as ordinate and β as abscissa and with different curve sheets corresponding to

fixed values of ρ, there might be some crossing of individual curves; that is, for certain values of β two distinct modes might have the same frequency. Clearly this would confuse the significance of characterizing different modes simply by ordinal number in the order of increasing frequency and would indicate that a more sophisticated interpolation is required. However, no such graphs were prepared and there do not seem to be enough values of β represented in the graphs at hand to permit making replots. Thus it is suggested that some future investigator might find it interesting to employ the computer programs, incorporating slight modifications, to obtain such graphs and thus investigate the reasons for inaccurate interpolation for the higher modes.

IV. ACCURACY AND ASSURANCE OF SOLUTION

The allowable error permitted in determining the roots is 10^{-6} and this was assumed to be satisfactory for practical purposes. The allowable error can be changed from 10^{-6} to any desired value by changing the value EPS in the computer programs. This can be done by changing the card 000090 in the program IN-PLANE and card 000100 in the program OUT-OF-PLANE.

Assurance of solution was established in the following ways.

1. In the case ρ is equal to zero, the ell-shaped configuration becomes a simple built-in beam and solution for this case can be compared with the classical solution.

Degenerate Case; $\rho = 0$			
Mode No.	Values of (PL)		
	Comparison Values*	In - plane Solution	Out-of-plane Solution
1	4.7300407	4.7300289	4.7300411
2	7.8532046	7.8531849	7.8532047
3	10.9956078	10.9955803	10.9956081
4	14.1371655	14.1371302	14.1371655
5	17.2787597	17.2787165	17.2787594
6	20.4203522	20.4203012	20.4203526
7	23.5619449	23.5618858	23.5619449
8	26.7035376	26.7034814	26.7035412
9	29.8451302	29.8453738	29.8451861
10	32.9867229	32.9890104	32.9866973

*Obtained by computer solution of the equation Cosh(PL) Cos(PL) = 1.

In equation (2-21) when ρ is set to zero the value of the determinant is zero regardless of the value of (PL). Therefore ρ was taken to be 0.00001.

In out-of-plane case for the larger values of β, pure torsional modes appeared and this can be seen from Figures (3-5) and (3-6). The values of (PL) given in the above table were taken when β is equal to 0.001.

2. Consider the cases shown in Figure 4-1.

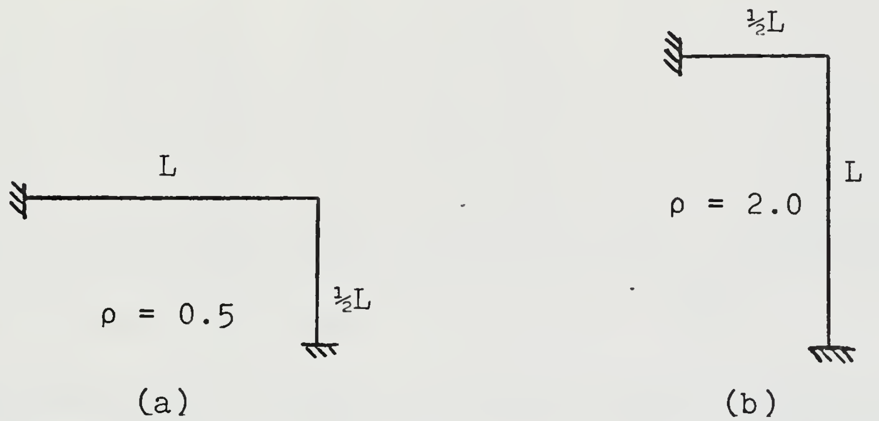


Figure 4-1. Cases Having Reciprocal Values of ρ.

The configurations shown in the above figure are identical, therefore two systems must have the same frequencies. Using equation (2-22) as

$$(PL)^2_{ai}\sqrt{\frac{EI}{\mu L^4}} = (PL)^2_{bi}\sqrt{\frac{EI}{\mu(\frac{L}{2})^4}}$$

then

$$\frac{(PL)_{ai}}{(PL)_{bi}} = 2$$

Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	4.3755336	2.1877668	2.000000000
2	7.2490132	3.6245066	2.000000000
3	8.7635958	4.3817979	2.000000000
4	10.707676	5.3538381	1.999999999
5	13.540341	6.7701705	2.000000001
6	15.024013	7.5120064	1.999999999
7	16.987974	8.4939872	2.000000000
8	19.8231918	9.9115959	2.000000000
9	21.3082077	10.6541038	2.000000001
10	23.2712829	11.6356415	2.000000001

For out-of-plane solution in the case $\beta = 0.001$

Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	2.9832348	1.4916187	1.999998209
2	4.7014996	2.3507534	1.999996916
3	7.3772076	3.6886158	1.999993485
4	8.8416393	4.4208958	1.999965571
5	10.8159008	5.4080044	1.999980037
6	13.6035633	6.8018597	1.999977062
7	15.0968311	7.5488148	1.999894233
8	17.0625606	8.5315098	1.999946210
9	19.8673419	9.9339245	1.999948965
10	21.3568783	10.6796268	1.999777575

3. In the case ρ is equal to one, it is expected that among the in-plane solutions will be many for which the moment at the joint vanishes. Thus, these cases also pertain to the clamped-pinned simple beam. Now two clamped-pinned simple beams of different lengths, ℓ and ℓ^*, may have matching frequencies in their spectra. That is, for example, the second frequency for one may equal the first frequency for the other. Specifically the following case was considered.

The third frequency for an equal-leg ell is equal to the second frequency of a clamped-pinned beam of length ℓ, namely

$$\omega_3 = (PL)_3^2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_3 = 7.06858275$$

The first frequency of an equal-leg ell of length ℓ^* is equal to the first frequency of clamped-pinned beam of length ℓ^*, namely

$$\omega_1 = (PL)_1^2 \sqrt{\frac{EI}{\mu \ell^{*4}}}$$

where

$$(PL)_1 = 3.92660231$$

The ratio ρ was determined so that then the frequencies are equal, thus

$$\rho = \frac{\ell^*}{\ell} = \frac{(PL)_1}{(PL)_3} = 0.55550065$$

Then performing an analysis for an ell-shaped configuration with the value of ρ, found above, the second frequency is indeed,

$$\omega_2 = (PL)_2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_2 = 7.068384686$$

This value shows close agreement with the value

$$(PL)_3 = 7.0685275$$

given above.

APPENDIX A

DESCRIPTION OF PROGRAMS

A. GENERAL REMARKS

Programs IN-PLANE and OUT-OF-PLANE are FORTRAN-IV language digital computer programs designed for use in the vibration analysis of an ell-shaped uniform filament with clamped ends.

Both programs employ double precision arithmetic. Subroutines are provided in binary form. Trigometric and hyperbolic functions, available in the IBM-360/67 library, were used.

B. PROGRAM STRUCTURE

Each program consists of a four section main program and two subroutines. The function of each section of the main program and of each subroutine is given below.

1. Main program subsections and their functions.

INPUT	Controls read-in of data and allocates storage locations for arrays and subscripted variables. The allowable error for determination of roots is specified in this section.
ITERATION AND EVALUATION OF ROOTS	In this section the roots of the determinant are located in an interval by searching the sign changes of the determinant and then the method of successive bisectioning is used to evaluate the roots. The maximum number of iterations was limited to three hundred and this was assumed sufficient. If the number of iterations becomes as large as three

hundred the process terminates and the approximate root is tagged to indicate a possible inaccurate result.

OUTPUT — Controls output format.

GRAPH — Controls the graph output and uses the DRAW subroutine provided in the IBM-360/67 library.

2. Subroutines and their functions.

BRODE — In program IN-PLANE equation (2-21) is stored and in program OUT-OF-PLANE equation (2-30) is stored. Evaluates each element of the determinant and obtains the value of the determinant by calling subroutine DTERM.

DTERM — This subroutine computes the determinant of a matrix of real numbers by Gauss' method of elimination with row pivoting.

C. PROGRAM NOMENCLATURE

A	Elements of determinants.
B	Checks sign of determinant and if there is a sign change moves control to the method of successive bisectioning.
BB	Values of β in subroutine BRODE.
BBB	Checks the acceptability of roots.
BETA	Value of β.
CEM	Single precision version of RO.
DET	Value of determinant.
EPS	Allowable error in roots.
F	Value of determinant.
N	Index of modes (Ordinal number).
P	Values of (PL) used to search interval.
R3, R4	Values of determinant in the method of successive bisectioning.
RO	Values of ρ.

ROOT	Roots of determinant (PL).
RR	Corresponds to RO in subroutine BRODE.
SEV	Single precision version of ROOT.
X	Middle of interval.
XX1; XX2	Upper and lower limits of interval containing the root.
YY	Values of P, X, XX1 and XX2 in subroutine BRODE.

APPENDIX B

INSTRUCTIONS FOR USE OF PROGRAMS

A. DATA REQUIRED

1. Data required for program IN-PLANE

a. Number of ρ's (NRO).

b. Number of modes sought (MODE).

c. Graph control (NGRAPH).

d. Values of ρ (RO).

e. Graph title (ITITLE).

2. Data required for program OUT-OF-PLANE

a. Number of ρ's (NRO)

b. Number of β's (NBETA).

c. Number of modes sought (MODE).

d. Graph control (NGRAPH).

e. Values of β (BETA).

f. Values of ρ (RO).

g. Graph title (ITITLE).

B. DATA DECK ASSEMBLY

1. Program IN-PLANE

The data deck is ordered as follows:

a. Problem statement card.

b. Values of ρ card(s).

c. Graph title cards.

2. Program OUT-OF-PLANE

The data deck is ordered as follows:

a. Problem statement card.

b. Values of β card(s).

c. Values of ρ card(s).

d. Graph title cards.

C. LIMITING THE NUMBER OF ITERATIONS

In the programs the maximum number of iterations was set to 300. If the number of iterations reaches 300 for any root, the current value is assumed as a solution but in the output it is marked as ITER. EXCEEDED 300. A user either accepts the root or changes the cards 000430 in program IN-PLANE and 000460 in program OUT-OF-PLANE by changing 300 to any desired value.

D. GRAPH

A user can get graphical output by option. DRAW subroutine, available in IBM-360/67 library was used and three CALL DRAW statements control first, intermediate and last curves to be plotted on one graph. DRAW subroutine requires single precision arithmetic. Therefore variables CEM and SEV are introduced to change the values of RO and ROOT from double precision to single precision.

E. FORMAT OF DATA

1. Program IN-PLANE

a. Problem statement card.

The card has 3 fields under the control of following field specification.

3 I 3

Field names and the information each field conveys are given below in order.

Field names and content.

(1) NRO Number of ρ's

(2) MODE Number of modes sought.

(3) NGRAPH Code specifying whether there is to be graph output.
(NGRAPH = 0 No graph output)
(NGRAPH = 1 Graph will be output)

b. Values of ρ card(s).

Field specification 7F10.0.

If there are more than 7 different values of ρ, additional card(s) will be required with the same field specification.

c. Graph title cards.

The title of graph is read as data with the field specification 6A8. For the title two cards are required. The first card corresponds to the first line of the title and the second card corresponds to the second line.

2. Program OUT-OF-PLANE

a. Problem statement card.

The card has 4 fields under the following field specification

4 I 3

Field names are given below in order.

(1) NRO Number of ρ's.

(2) NBETA Number of β's.

(3) MODE Number of modes sought.

(4) NGRAPH Same as in program IN-PLANE

b. Values of β cards.

Field specification 7F10.0 and field name is BETA.

c. Values of ρ card(s).

Same as in program IN-PLANE.

d. Graph title cards.

Same as in program IN-PLANE.

APPENDIX C

FLOW DIAGRAMS

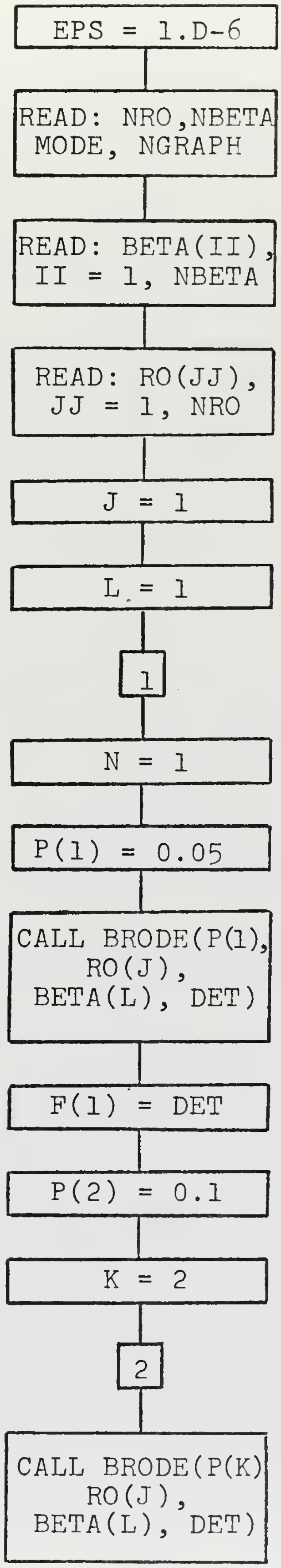
A. GENERAL

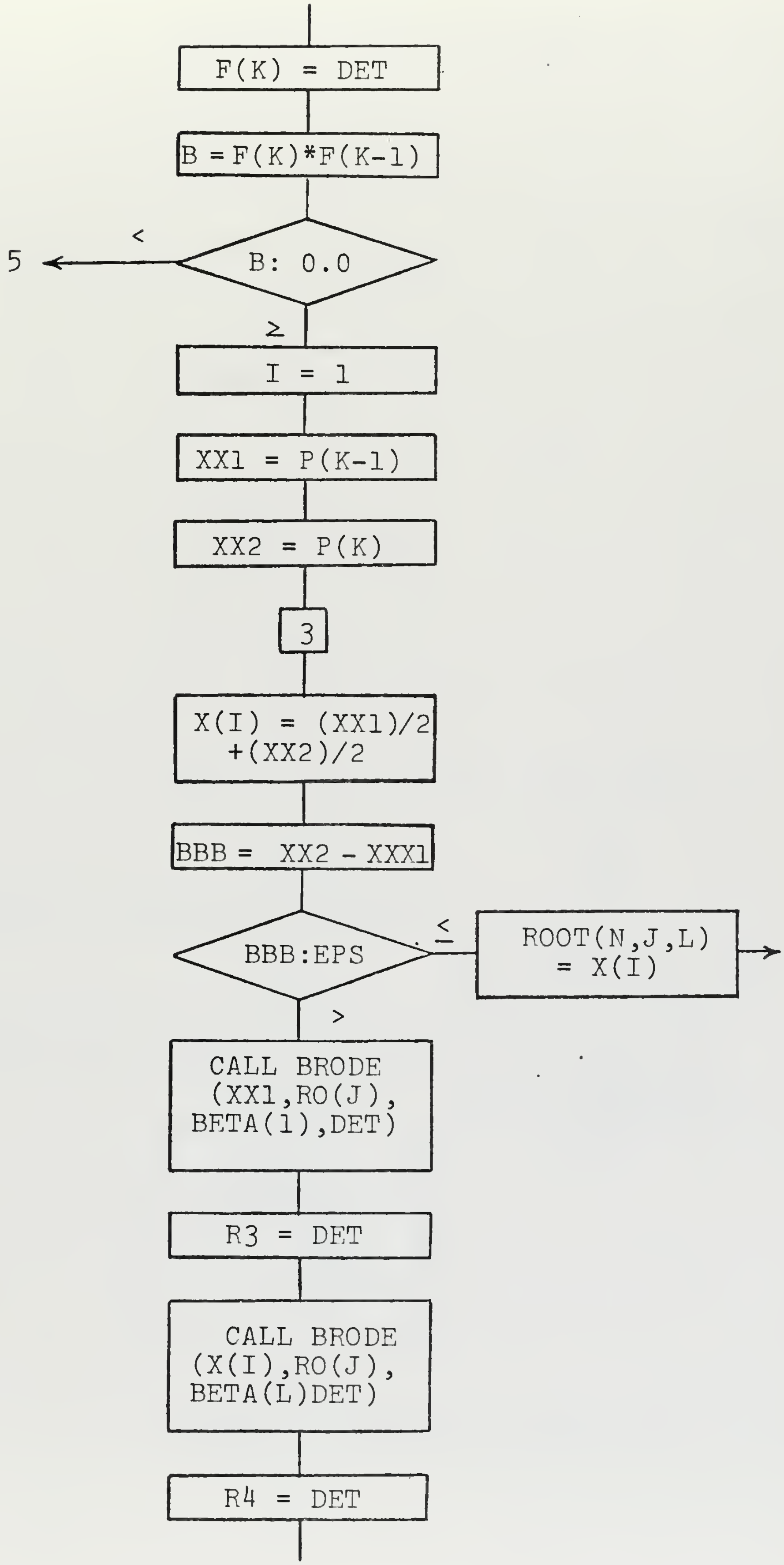
Flow diagram of the program OUT-OF-PLANE is given. This flow diagram includes all parts of the main program. Flow diagrams of subroutines BRODE and DTERM are not given and their functions were explained in Section A-2 in Appendix A.

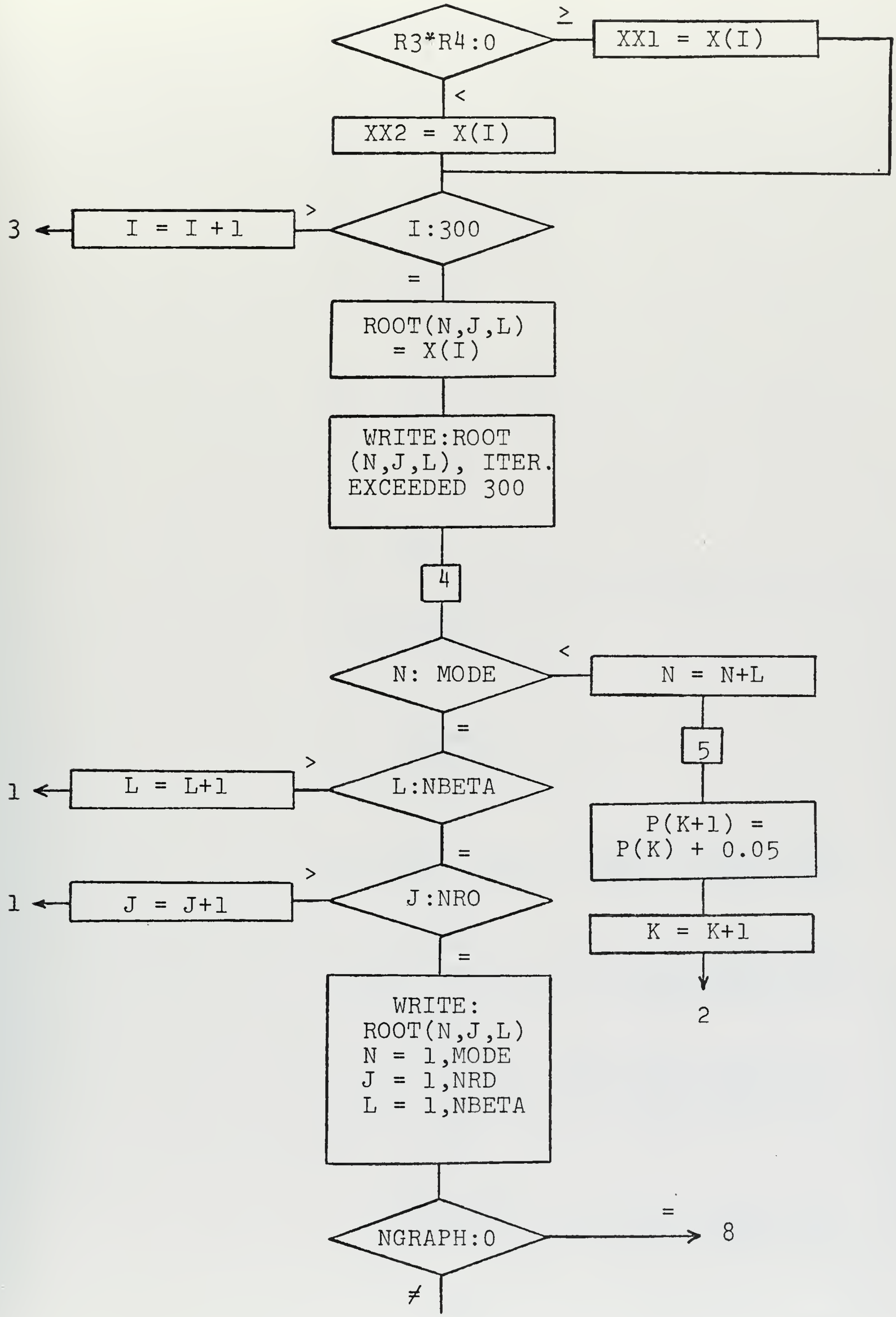
Program IN-PLANE is similar to OUT-OF-PLANE, therefore its flow diagram is not given.

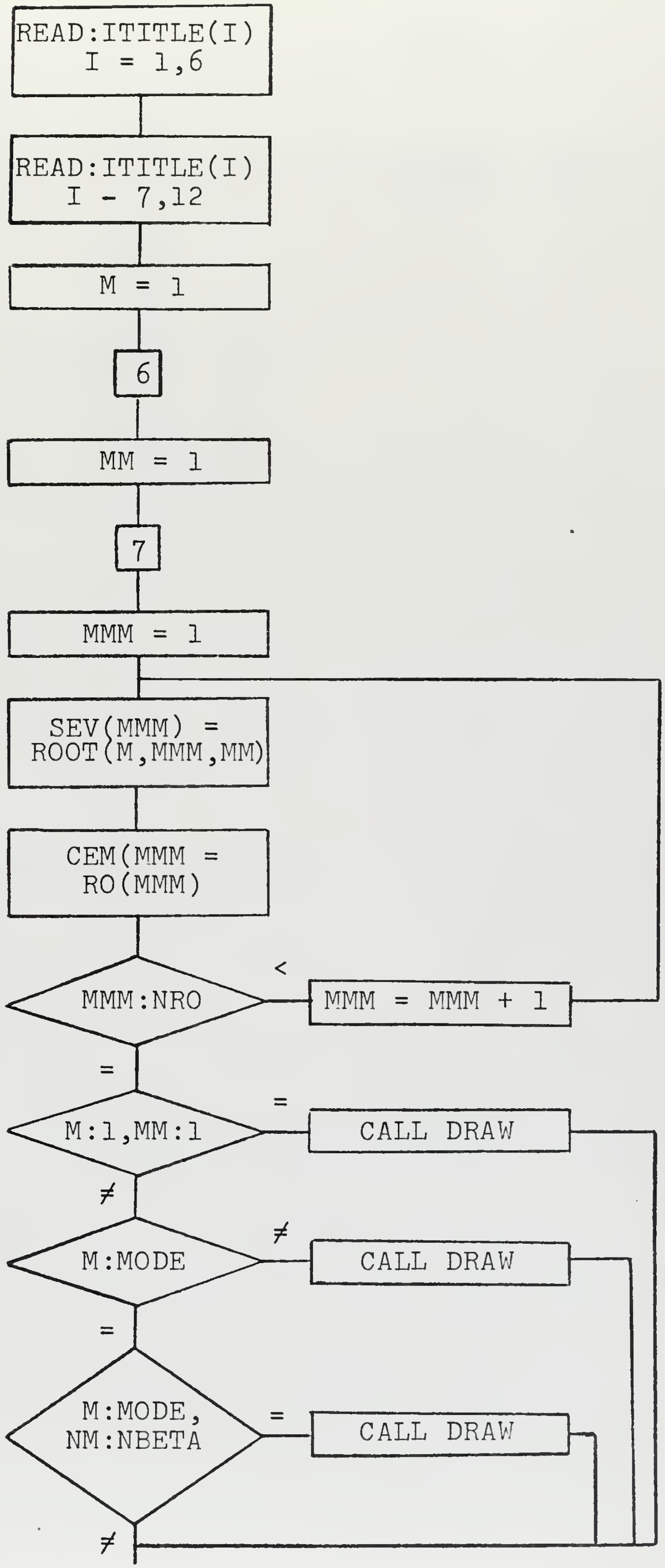
B. FLOW DIAGRAM OF PROGRAM OUT-OF-PLANE

Flow diagram follows.

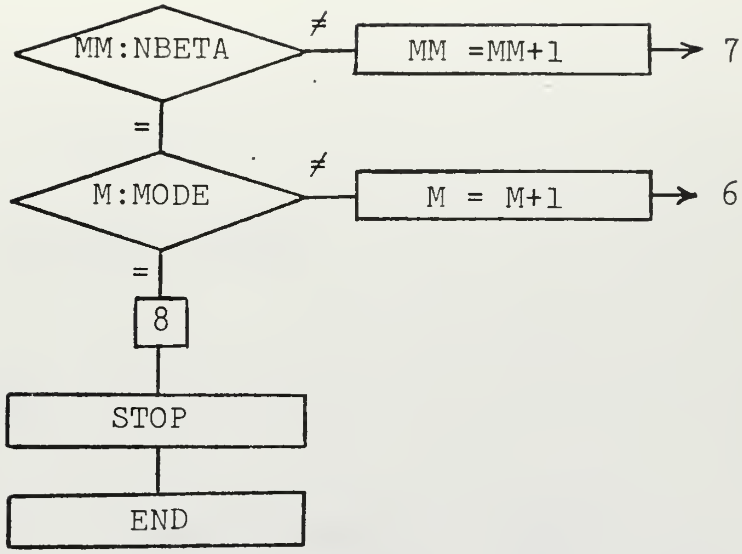

EPS = 1.D-6
READ: NRO,NBETA MODE, NGRAPH
READ: BETA(II), II = 1, NBETA
READ: RO(JJ), JJ = 1, NRO
J = 1
L = 1
1
N = 1
P(1) = 0.05
CALL BRODE(P(1), RO(J), BETA(L), DET)
F(1) = DET
P(2) = 0.1
K = 2
2
CALL BRODE(P(K) RO(J), BETA(L), DET)


F(K) = DET
B = F(K)*F(K-1)
B: 0.0
<
5
≥
I = 1
XX1 = P(K-1)
XX2 = P(K)
3
X(I) = (XX1)/2 +(XX2)/2
BBB = XX2 - XXX1
BBB:EPS
≤
ROOT(N,J,L) = X(I)
>
CALL BRODE (XX1,RO(J), BETA(1),DET)
R3 = DET
CALL BRODE (X(I),RO(J), BETA(L)DET)
R4 = DET


R3*R4:0
≥
XX1 = X(I)
<
XX2 = X(I)
3
I = I +1
>
I:300
=
ROOT(N,J,L) = X(I)
WRITE:ROOT (N,J,L), ITER. EXCEEDED 300
4
N: MODE
<
N = N+L
=
5
1
L = L+1
>
L:NBETA
P(K+1) = P(K) + 0.05
=
1
J = J+1
>
J:NRO
K = K+1
=
2
WRITE: ROOT(N,J,L) N = 1,MODE J = 1,NRD L = 1,NBETA
NGRAPH:0
=
8
≠



READ:ITITLE(I) I = 1,6
READ:ITITLE(I) I - 7,12
M = 1
6
MM = 1
7
MMM = 1
SEV(MMM) = ROOT(M,MMM,MM)
CEM(MMM = RO(MMM)
MMM:NRO
<
MMM = MMM + 1
=
M:1,MM:1
=
CALL DRAW
≠
M:MODE
≠
CALL DRAW
=
M:MODE, NM:NBETA
=
CALL DRAW
≠


MM:NBETA
≠
MM =MM+1
7
=
M:MODE
≠
M = M+1
6
=
8
STOP
END

APPENDIX D

PROGRAM LISTING

A. INDEX

1. Program IN-PLANE

Main program:

Subroutines:

2. Program OUT-OF-PLANE

Main program:

Subroutines:

B. LISTING

```
      PROGRAM IN-PLANE

      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 SEV,CEM
      REAL*8 ITITLE(12)
      REAL*4LABEL/4H    /
      DIMENSION P(900),X(350),RO(60),ROOT(15,60),SEV(60),CEM(60),F(900)
C
C     INPUT
C
      EPS=1.0D-8
      READ(5,1) NRO,MODE,NGRAPH
    1 FORMAT(3I3)
      READ(5,2) (RO(JJ),JJ=1,NRO)
    2 FORMAT(7F10.0)
C
C     ITERATION AND EVALUATION OF ROOTS
C
      DO 500 J=1,NRO
      N=1
      P(1)=0.05D0
      CALL BRODE(P(1),RO(J),DET)
      F(1)=DET
      P(2)=0.1D0
      DO 110 K=2,800
      CALL BRODE(P(K),RO(J),DET)
      F(K)=DET
      B=F(K)*F(K-1)
      IF(B.GT.0.0D0) GO TO 20
      I=1
      XX1=P(K-1)
      XX2=P(K)
    9 X(I)=(XX1+XX2)/2.0D0
      BBB=DABS(XX2-XX1)
      IF(BBB.LE.EPS) GO TO 12
      CALL BRODE(XX1,RO(J),DET)
      R3=DET
      CALL BRODE(X(I),RO(J),DET)
      R4=DET
      IF(R3.GT.0.0D0.AND.R4.GT.0.0D0) GO TO 10
      IF(R3.LT.0.0D0.AND.R4.LT.0.0D0) GO TO 10
      XX2=X(I)
      GO TO 11
   10 XX1=X(I)
   11 IF(I.EQ.300) GO TO 14
      I=I+1
      GO TO 9
   12 ROOT(N,J)=X(I)
```

```
      GO TO 16
   14 ROOT(N,J)=X(I)
      WRITE(6,15) N,ROOT(N,J)
   15 FORMAT('0',47X,I3,10X,F20.16,8HITER=300)
   16 IF(N.EQ.MODE) GO TO 500
      N=N+1
   20 P(K+1)=P(K)+0.05D0
  110 CONTINUE
  500 CONTINUE
C
C     OUTPUT
C
      WRITE(6,1200)
 1200 FORMAT('1',43X,12HCETIN  DERIN,10X,14HDECEMBER  1970)
      WRITE(6,1300)
 1300 FORMAT('0',116H***************************************************
     1**************************************************************** )
      WRITE(6,1400)
 1400 FORMAT('0',49X,19HIN PLANE VIBRATIONS)
      DO 30 L=1,NRO
      WRITE(6,1500) RO(L)
 1500 FORMAT('0',53X,3HRO=,F7.5)
      WRITE(6,1600)
 1600 FORMAT(' ',50X,16H****************)
      WRITE(6,1700)
 1700 FORMAT('0',47X,4HMODE,16X,2HPL)
      DO 31 I=1,MODE
      WRITE(6,13) I,ROOT(I,L)
   13 FORMAT('0',47X,I3,10X,F20.16)
   31 CONTINUE
   30 CONTINUE
C
C     GRAPH
C
      IF(NGRAPH.EQ.0) GO TO 7000
      READ(5,8) (ITITLE(I),I=1,6)
      READ(5,8) (ITITLE(I),I=7,12)
    8 FORMAT(6A8)
      DO 800 MM=1,MODE
      DO 700 M=1,NRO
      SEV(M)=ROOT(MM,M)
      CEM(M)=RO(M)
  700 CONTINUE
      IF(MM.EQ.1) GO TO 4100
      IF(MM.GT.1.AND.MM.LT.MODE) GO TO 4200
      IF(MM.EQ.MODE) GO TO 4300
 4100 CALL DRAW(NRO,CEM,SEV,1,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
```

```
      GO TO 150
 4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
      GO TO 150
 4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
  150 WRITE(6,4400) LAST
 4400 FORMAT(1H0,5X,'LAST=',I2)
  800 CONTINUE
 7000 STOP
      END


      SUBROUTINE BRODE(YY,RR,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4)
      Y=0.5D0
      Z=0.0D0
      U=YY*RR
      A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
      A(1,2)=Z
      A(1,3)=Y*(DSINH(YY)-DSIN(YY))
      A(1,4)=Z
      A(2,1)=Z
      A(2,2)=Y*(DCOSH(U)-DCOS(U))
      A(2,3)=Z
      A(2,4)=Y*(DSINH(U)-DSIN(U))
      A(3,1)=Y*(DSINH(YY)+DSIN(YY))
      A(3,2)=-Y*(DSINH(U)+DSIN(U))
      A(3,3)=A(1,1)
      A(3,4)=-A(2,2)
      A(4,1)=Y*(DCOSH(YY)+DCOS(YY))
      A(4,2)=Y*(DCOSH(U)+DCOS(U))
      A(4,3)=A(3,1)
      A(4,4)=-A(3,2)
      CALL DTERM(4,A,DET,4)
      RETURN
      END


      SUBROUTINE DTERM(N,A,D,M)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(M,M)
      DC=1.D0
   10 DO 34 L=1,N
      KP=0
      Z=0.0D0
      DO 12 K=L,N
```

```
      IF(Z-DABS(A(K,L)))11,12,12
   11 Z=DABS(A(K,L))
      KP=K
   12 CONTINUE
      IF(L-KP) 13,20,20
   13 DO 14  J=L,N
      Z=A(L,J)
      A(L,J)=A(KP,J)
   14 A(KP,J)=Z
      DD=-DD
   20 IF(L-N) 31,40,40
   31 LP1=L+1
      DO 34  K=LP1,N
      IF(A(K,L)) 32,34,32
   32 RATIO=A(K,L)/A(L,L)
      DO 33  J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
   34 CONTINUE
   40 DO 41  K = 1,N
   41 DD=DD*A(K,K)
      D=DD
      RETURN
      END
```

```
      PROGRAM OUT-OF-PLANE
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 SEV,CEM
      REAL*8 ITITLE(12)
      REAL*4 LABEL/4H    /
      DIMENSION P(900),X(350),RO(60),ROOT(15,60,5),BETA(5),SEV(60),CEM(
     160),F(900)
C
C     INPUT
C
      EPS=1.0D-6
      READ(5,1) NRO,NBETA,MODE,NGRAPH
    1 FORMAT(4I3)
      READ(5,2) (BETA(II),II=1,NBETA)
      READ(5,2) (RO(JJ),JJ=1,NRO)
    2 FORMAT(7F10.0)
C
C     ITERATION AND EVALUATION OF ROOTS
C
      DO 19 J=1,NRO
      DO 18 L=1,NBETA
      N=1
      P(1)=0.05D0
      CALL BRODE(P(1),RO(J),BETA(L),DET)
      F(1)=DET
      P(2)=0.1D0
      DO 17 K=2,850
      CALL BRODE(P(K),RO(J),BETA(L),DET)
      F(K)=DET
      B=F(K)*F(K-1)
      IF(B.GT.0.0D0) GO TO 20
      I=1
      XX1=P(K-1)
      XX2=P(K)
    9 X(I)=(XX1+XX2)/2.0D0
      BBB=DABS(XX2-XX1)
      IF(BBB.LE.EPS) GO TO 12
      CALL BRODE(XX1,RO(J),BETA(L),DET)
      R3=DET
      CALL BRODE(X(I),RO(J),BETA(L),DET)
      R4=DET
      IF(R3.GT.0.0D0.AND.R4.GT.0.0D0) GO TO 10
      IF(R3.LT.0.0D0.AND.R4.LT.0.0D0) GO TO 10
      XX2=X(I)
      GO TO 11
   10 XX1=X(I)
   11 IF(I.EQ.300) GO TO 14
```

```
      I=I+1
      GO TO 9
   12 ROOT(N,J,L)=X(I)
      GO TO 16
   14 ROOT(N,J,L)=X(I)
      WRITE(6,15) N,ROOT(N,J,L)
   15 FORMAT('0',47X,I2,10X,F19.16,18HITER. EXCEEDED 300)
   16 IF(N.EQ.MODE) GO TO 18
      N=N+1
   20 P(K+1)=P(K)+0.05D0
   17 CONTINUE
   18 CONTINUE
   19 CONTINUE
C
C     OUTPUT
C
      WRITE(6,3)
    3 FORMAT('1',45X,12HCETIN  DERIN,10X,14HDECEMBER  1970)
      WRITE(6,4)
    4 FORMAT('0',116H**************************************************
     1**************************************************************** )
      WRITE(6,5)
    5 FORMAT('0',51X,23HOUT OF PLANE VIBRATIONS)
      DO 5200 IJ=1,NRO
      DO 5100 IL=1,NBETA
      WRITE(6,6) RO(IJ),BETA(IL)
    6 FORMAT('0',49X,3HRO=,F7.5,5X,5HBETA=,F7.5)
      WRITE(6,7)
    7 FORMAT(' ',46X,33H*********************************)
      WRITE(6,8)
    8 FORMAT('0',46X,4HMODE,16X,2HPL)
      DO 5000 IN=1,MODE
      WRITE(6,13) IN,ROOT(IN,IJ,IL)
   13 FORMAT('0',47X,I2,10X,F19.16)
 5000 CONTINUE
 5100 CONTINUE
 5200 CONTINUE
C
C     GRAPH
C
      IF(NGRAPH.EQ.0) GO TO 7000
      READ(5,4600) (ITITLE(I),I=1,6)
      READ(5,4600) (ITITLE(I),I=7,12)
 4600 FORMAT(6A8)
      DO 3300 M=1,MODE
      DO 3200 MM=1,NBETA
      DO 3100 MMM=1,NRO
      SEV(MMM)=ROOT(M,MMM,MM)
```

```
      CEM(MMM)=RO(MMM)
 3100 CONTINUE
      IF(M.EQ.1.AND.MM.EQ.1) GO TO 4100
      IF(M.NE.MODE) GO TO 4200
      IF(M.EQ.MODE.AND.MM.EQ.NBETA) GO TO 4300
 4100 CALL DRAW(NRO,CEM,SEV,1,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
      GO TO 4400
 4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
      GO TO 4400
 4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
     1LAST)
 4400 WRITE(6,4500) LAST
 4500 FORMAT(1H0,5X,'LAST=',I2)
 3200 CONTINUE
 3300 CONTINUE
 7000 STOP
      END


      SUBROUTINE BRODE(YY,RR,BB,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6)
      Y=0.5D0
      Z=0.0D0
      U=YY*RR
      W=BB*YY**2
      V=W*RR
      A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
      A(1,2)=-Y*(DCOSH(U)-DCOS(U))
      A(1,3)=Z
      A(1,4)=Z
      A(1,5)=Y*(DSINH(YY)-DSIN(YY))
      A(1,6)=-Y*(DSINH(U)-DSIN(U))
      A(2,1)=Z
      A(2,2)=Y*(DSINH(U)+DSIN(U))
      A(2,3)=-DSIN(W)*1.3D0/BB
      A(2,4)=Z
      A(2,5)=Z
      A(2,6)=-A(1,2)
      A(3,1)=Y*(DSINH(YY)+DSIN(YY))
      A(3,2)=Z
      A(3,3)=Z
      A(3,4)=DSIN(V)*1.3D0/BB
      A(3,5)=A(1,1)
      A(3,6)=Z
      A(4,1)=A(1,5)
```

```
      A(4,2)=-A(1,6)
      A(4,3)=Z
      A(4,4)=Z
      A(4,5)=Y*(DCOSH(YY)+DCOS(YY))
      A(4,6)=Y*(DCOSH(U)+DCOS(U))
      A(5,1)=A(4,5)
      A(5,2)=Z
      A(5,3)=Z
      A(5,4)=-YY*DCOS(V)
      A(5,5)=A(3,1)
      A(5,6)=Z
      A(6,1)=Z
      A(6,2)=A(4,6)
      A(6,3)=YY*DCOS(W)
      A(6,4)=Z
      A(6,5)=Z
      A(6,6)=A(2,2)
      CALL DTERM(6,A,DET,6)
      RETURN
      END


      SUBROUTINE DTERM(N,A,D,M)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(M,M)
      DD=1.D0
   10 DO 34 L=1,N
      KP=0
      Z=0.0D0
      DO 12 K=L,N
      IF(Z-DABS(A(K,L)))11,12,12
   11 Z=DABS(A(K,L))
      KP=K
   12 CONTINUE
      IF(L-KP) 13,20,20
   13 DO 14 J=L,N
      Z=A(L,J)
      A(L,J)=A(KP,J)
   14 A(KP,J)=Z
      DD=-DD
   20 IF(L-N) 31,40,40
   31 LP1=L+1
      DO 34 K=LP1,N
      IF(A(K,L)) 32,34,32
   32 RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
   34 CONTINUE
```

```
40  DO 41  K = 1,N
41  DD=DD*A(K,K)
    D=DD
    RETURN
    END
```

APPENDIX E

PROGRAM VIPIPE

A. GENERAL

As a part of the present thesis investigation, attention was given to a digital computer program called VIPIPE which was originally written by Fink [1] and later modified and augmented by Kim [2]. This program provides solutions for in-plane and out-of-plane vibration of a single plane piping configuration having quite general geometry (except, of course, for its limitation to a single plane). It was desired to convert this program from a FORTRAN 63 program for use with CDC-1604 computer formerly employed at the Naval Postgraduate School, to a FORTRAN IV program for the IBM 360/67 presently in use, in order to provide an independent means of checking the results obtained as indicated in the body of this thesis and, also, because it would be useful to have a program of such generality available. For details of the theory of VIPIPE, refer to the theses written by Fink and Kim.

Because of the limitations on word length in IBM 360/67, the converted program, which was called VIPIPE-1, has been written to employ double precision arithmetic. Certain improvements have been made in the capability to produce graphical output.

The effort to make this conversion has been found to be much greater than was anticipated. It seems that CDC-1604 computer is a more "forgiving" machine than is the IBM 360/67. In checking out the new program certain bugs came to light which were actually present in the earlier version but which apparently did not cause trouble. However, they represented some syntactical errors that resulted in failure on the IBM 360/67. In some cases it was clear how to modify the program so as to eliminate the errors but in other cases the difficulty was so deep in the logic that a correction could not be made without acquiring as deep a knowledge of the program as was possessed by original writers, Fink and Kim.

Thus it cannot be asserted that VIPIPE-1 has been successfully converted in every fine detail and it is recommended that additional efforts be made to assure the overall integrity of the program and to assess its limitations. It should be noted, however, that no gross errors have been found in any results obtained by use of the new program; the only question is that of the degree of accuracy that it is possible to obtain.

Comparisons of accuracy are difficult to make. In the first place, the word-length is different in the new program, and, other things being equal, one would expect the new program to provide better accuracy, and to maintain its integrity in the face of round-off error beyond the point where the earlier programs "bombed out." Thus, agreement

between Kim and Fink and disagreement with VIPIPE-1 does not mean that the former are "correct" and the latter is in error. Furthermore, Kim reports having discovered and corrected some bugs which he found in Fink's version. Also, there is some difficulty in understanding the precise details of some of the problems treated by Fink and Kim.

With this preface which admonishes the reader not to make superficial inferences from comparisons, in Section C some comparisons between results obtained by Kim and results obtained using VIPIPE are presented.

B. DATA DECK ASSEMBLY

The order of data deck is

1. Graph title data cards.
2. Problem statement card.
3. Boundary condition data card(s).
4. Branch joining angle data card(s).
5. Component extensive properties data card(s).
6. Component intensive properties data card(s).

Field specification and format of graph title cards are given in Section E-1-c of Appendix B and information for the remaining cards are given in [1].

C. SAMPLE PROBLEMS

Properties of piping systems analyzed in this section are given in [1]. Here, only numerical values of frequencies will be given.

1. System 1 (in-plane)

Mode No.	Comparison Values*	VIPIPE** Delta m. (D)	VIPIPE-1*** Delta m.
1	75.09994540	75.09994544	75.09994535
2	207.01589137	207.01589036	207.01589099
3	405.83391916	405.83393183	405.83391910
4	670.86408383	670.86310856	670.86408365
5	1002.15520225	1002.19978814	1002.15520252
6	1399.70436269		1399.70434917
7	1863.51172599		1863.51198134

*Sources of comparison frequencies for system 1 and system 2 are given in Section D.

**Frequencies are given in [1].

*** Frequencies are found by VIPIPE-1.

2. System 2 (in-plane)

Mode No.	Comparison Values	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	11.80213586	11.80213389	11.80213432
2	73.96272297	73.96272167	73.96272087
3	207.09776595	207.09776685	207.09776559
4	405.82896584	405.82894132	405.82896568
5	670.86435913	670.86200817	670.86435889
6	1002.15518770	1002.26833752	1002.15518758
7	1399.70436229	1399.08849534	1399.70437163

3. System 4 (in-plane)

Mode No.	VIPIPE M-D m. (D)	VIPIPE-1 M-D m.
1	66.69934902	67.31642169
2	171.22352538	172.86857155
3	335.35007083	339.09491357
4	535.64529414	544.08929568
5	773.44358487	794.57343086

4. System 5 (in-plane)

Mode No.	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	116.89479180	118.95185774
2	304.88354398	302.91288556
3	566.43670976	519.19656352

Mode No.	VIPIPE P.M. m.-C (D)	VIPIPE-1 P.M. m. -C
1		118.95185779
2		302.91288592
3		519.19656389
4	826.88281929	794.49003694
5	1126.83797508	1126.27858174
6	1666.58878192	1464.59530724
7	2099.52844030	1874.97923215

D. SOURCES OF COMPARISON FREQUENCIES FOR SYSTEM 1 AND SYSTEM 2

The governing fourth order equation of a homogeneous single component straight system

$$y^{IV} = \frac{\mu\omega^4}{EI} y \tag{E-1}$$

was solved to give

$$y = a\cos Px + b \cosh Px + c \sin Px + d \sinh Px$$

Upon applying boundary conditions for system 1* ($y = y' = 0$ at $x = 0$, $x = L$) and simplifying, it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = 1 \tag{E-2}$$

Upon applying the boundary conditions for system 2 ($y = y' = 0$ at $x = 0$, $y'' = y''' = 0$ at $x = L$), it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = -1 \tag{E-3}$$

The roots of equation E-2 and E-3, from which the natural frequencies of corresponding systems may easily be found, were obtained through iteration using a digital computer.

---

*System 1 is clamped-clamped end beam
System 2 is clamped-free end beam

E. PROGRAM LISTING

1. Index

Main program:

Subroutines:

2. Listing

```
      PROGRAM VIPIPE-1
C
C     INPUT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 CET,ZERGU
      REAL*8 ITITLE(12)
      REAL LABEL/'    '/
      REAL LABEL1/'ONE '/
      REAL LABEL2/'TWO '/
      REAL LABEL3/'THRE'/
      REAL LABEL4/'FOUR'/
      REAL LABEL5/'FIVE'/
      REAL LABEL6/'SIX '/
      REAL LABEL7/'SEVN'/
      REAL LABEL8/'EIT '/
      REAL LABEL9/'NINE'/
      REAL LABEL0/'TEN '/
      REAL LABELL/'LAST'/
      COMMON     AJT(50),AJ(50),R(50),AIX(50),AIY(50),PHI(20),SVI(6,1),
     1DT(50),D(50),DI(50),TL(50),Z(50),G(50),E(50),AMU(50),THETA(50),RHO
     2(50),OMEGAM(20),DEL(600),OMEGA(600),A(6,6),B(6,6),U(6,6),CLX(50),C
     3LY(50),CLZ(50),CTX(50),CTY(50),CTZ(50),V(6,3),VV(6,3),UU(6,3),SVE(
     46,1),SV(22),SVIP(6,22),SVO(22),SVOP(6,22),NNBR(50),BAMU(50),THETR(
     520),P(20),YY(3,3,23),R3(3,3),XX(3,3),EDL(600),ZERGU(600),CET(600)
      SEVIM=1.0D0
      EEVIN=0.0D0
      NM=0
      JJ=1
      READ(5,2001) (ITITLE(I),I=1,6)
      READ (5,2001) (ITITLE(I),I=7,12)
 2001 FORMAT(6A8)
  450 READ(5,1500) NS,NBR,IOP,HM,AMYK,SD,RI,OMEGAG,OMEGAI,GDABC,BRC,
     1GRAPH,REM,PMM,HL
 1500 FORMAT(3I2,4F3.0,2F8.0,4F2.0,2F4.0)
      IF(NS) 600,600,601
  601 NMBR=NBR+2
      IF(IOP-1) 1501,1501,1506
 1501 READ(5,1502)  (SV(I),I=1,NMBR)
 1502 FORMAT(22F3.0)
      DO 1505 J=1,NMBR
      IF(SV(J)) 1503,1503,1505
 1503 READ(5,1504) (SVIP(I,J),I=1,6)
 1504 FORMAT(6F2.0)
 1505 CONTINUE
      IF(IOP) 1506,1506,1523
 1506 READ(5,1507) (SVO(I),I=1,NMBR)
```

```
1507 FORMAT(22F3.0)
     DO 1510 J=1,NMBR
     IF(SVO(J)) 1508,1508,1510
1508 READ(5,1509) (SVOP(I,J),I=1,6)
1509 FORMAT(6F2.0)
1510 CONTINUE
1523 IF(NBR) 1513,1513,1511
1511 READ(5,1512) (PHI(I),I=1,NBR)
1512 FORMAT(10F7.3)
1513 READ(5,1514) (D(I),DI(I),RHO(I),TL(I),THETA(I),CLX(I),CLY(I),CLZ
    1(I),CTX(I),CTY(I),CTZ(I),NNBR(I),I=1,NS)
1514 FORMAT(3F5.2,2F6.2,6F7.2,I2)
     READ(5,1515) AAMU,AE,AG,DD
1515 FORMAT(F7.3,2E10.2,F2.0)
     IF(DD) 1516,1516,1518
1516 DO 1517 I=1,NS
     AMU(I)=AAMU
     E(I)=AE
1517 G(I)=AG
     GO TO 1520
1518 AMU(1)=AAMU
     E(1)=AE
     G(1)=AG
     READ(5,1519) (AMU(I),E(I),G(I),I=2,NS)
1519 FORMAT(F7.3,2E10.2)
1520 IF(OMEGAI)1121,1121,1521
1121 OMEGAI=0.01D0
1521 OMEGA(1)=OMEGAI
     AMYK=AMYK-SEVIM
     OMEGAO=OMEGAI
     FFAC=0.5D0
     L=1
     M=1
     MN=0
     AA=EEVIN
     BB=EEVIN
     CC=EEVIN
     FF=EEVIN
     GG=EEVIN
     RR=EEVIN
     KK=0
     PP=EEVIN
     KA=0
     KKK=0
     AAMU=EEVIN
     EE=EEVIN
     AL=EEVIN
     AD=EEVIN
```

```
      NNN=NBR+1
      IF(PMM*0.1D0-SEVIM) 6091,6092,6093
 6091 PM=-SEVIM
      GO TO 7000
 6092 PM=EEVIN
      GO TO 7000
 6093 IF(PMM*0.01D0-SEVIM) 6094,6095,6096
 6094 PM=-SEVIM
      GO TO 7000
 6095 PM=SEVIM
      GO TO 7000
 6096 IF(PMM*0.01D0-1.1D0) 6097,6098,6099
 6097 PM=-SEVIM
      GO TO 7000
 6098 PM=EEVIN
      GO TO 7000
 6099 PM=-SEVIM
 7000 PPM=PM
      IF(PM) 7073,7073,7001
 7001 IF(REM-SEVIM) 7071,7072,7071
 7071 SS=-SEVIM
      GO TO 7073
 7072 SS=EEVIN
 7073 DO 1522 I=1,NS
      DT(I)=DI(I)
      BAMU(I)=AMU(I)
      THETR(I)=THETA(I)
 1522 DI(I)=D(I)-2.0D0*DI(I)
C
C
C     INVARIANTS
C
      DO 108 I=1,NS
      IF(CLX(I)+CLY(I)+CLZ(I)+CTX(I)+CTY(I)+CTZ(I)) 108,101,108
  101 AMU(I)=AMU(I)*3.141592654D0*(D(I)**2-DI(I)**2)/(6912.0D0*32.17D0
     1*12.0D0)
      IF(RHO(I)) 102,103,102
  102 THETA(I)=THETA(I)*3.141592654D0/180.0D0
      TL(I)=THETA(I)*DABS(RHO(I))
  103 AJT(I)=3.141592654D0*(D(I)**4-DI(I)**4)/32.0D0
      AJ(I)=AJT(I)/2.0D0
      R(I)=SEVIM/(AJ(I)*E(I))
      AIX(I)=DSQRT((D(I)**2+DI(I)**2)/8.0D0)
      AIY(I)=AIX(I)/1.414214D0
      IF(AMYK) 104,105,104
  104 Z(I)=SEVIM
      IF(RHO(I)) 105,106,105
  105 HHM=HM
```

```
      HMH=EEVIN
      CALL SUBSEC (RHO(I),THETA(I),D(I),TL(I),HM,Z(I),OMEGAI,HMH)
      HM=HHM
  106 IF(NNBR(I)) 108,107,108
  107 AL=AL+TL(I)
      AD=AD+D(I)*TL(I)
      EE=EE+E(I)*TL(I)
      AID=AID+DI(I)*TL(I)
      AAMU=AAMU+AMU(I)*TL(I)
  108 CONTINUE
      AD=AD/AL
      AID=AID/AL
      AFR=3.141592654D0/64.0D0*(AD**4-AID**4)
      FR=4.73004019D0**2*DSQRT(EE*AFR/(AAMU*AL**4))
      ADOM=FR/4.0D0
      DOM=ADOM
      CR=ADOM*0.001D0
      BDOM=ADOM
      BR=CR
      ERM=REM
      DO 109 II=1,NBR
  109 PHI(II)=PHI(II)*3.141592654D0/180.0D0
      IF(IOP-1) 110,999,998
  110 MN=1
C
C
C     CONTROL IN PLANE
C
  999 P1=SEVIM
      P2= SEVIM
      D1=EEVIN
      D2=EEVIN
      Q1=SEVIM
      Q2=SEVIM
      Q3=SEVIM
      B1=EEVIN
      B2=EEVIN
      B3=EEVIN
      FINK=EEVIN
      IF(SV(1)) 1011,1021,1011
 1011 CALL STAVEC (SV(1),1,SVI)
      GO TO 1041
 1021 DO 1031 J=1,6
 1031 SVI(J,1)=SVIP(J,1)
 1041 IF(SV(NMBR)) 1051,1061,1051
 1051 CALL STAVEC(SV(NMBR),1,SVE)
      GO TO 1081
 1061 DO 1071 J=1,6
```

```
1071 SVE(J,1)=SVIP(J,NMBR)
1081 JK=0
     IF(NBR) 1000,1000,1085
1085 DO 1087 N=2,NNN
     IF(SV(N)) 1087,1087,1086
1086 CALL STAVEC(SV(N),N,SVIP)
1087 CONTINUE
1000 N=2
     II=1
     HH=EEVIN
     DO 1001 I=1,NS
     MAT=0
     BC=EEVIN
     P(I)=CLX(I)+CLY(I)+CTZ(I)
     IF(P(I)) 37,38,37
  37 CALL HANGER(CLX(I),CLY(I),CTZ(I),U)
     GO TO 62
  38 IF(AMYK) 39,41,39
  39 IF(RHO(I)) 40,42,40
  40 IF(Z(I)) 43,45,43
  41 IF(RHO(I)) 40,48,40
  42 IF(Z(I)) 46,47,46
  43 CALL CFIELD(R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),E(I),A)
  44 CALL POINT(AMU(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)
     CALL MATMUL(A,B,U)
     MAT=1
     GO TO 62
  45 CALL STIFCO(THETA(I),RHO(I),U)
     GO TO 62
  46 CALL DISTM(AMU(I),AIY(I),OMEGA(L),TL(I),R(I),D(I),DI(I),G(I),SD,
    1RI,E(I),FFAC,U)
     GO TO 62
  47 CALL RIGID(AMU(I),TL(I),OMEGA(L),AIY(I),RI,U)
     GO TO 62
  48 IF(Z(I)) 49,47,49
  49 CALL SFIELD(DI(I),TL(I),R(I),Z(I),G(I),SD,D(I),E(I),FFAC,A)
     GO TO 44
  62 IF(NNBR(I)) 81,80,81
  80 IF(I-1) 63,79,63
  79 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
     GO TO 63
  63 CALL FINMAT(U,UU,V,MAT,A,Z(I))
     DO 64 J=1,6
     DO 64 K=1,3
  64 UU(J,K)=V(J,K)
     IF(PM) 88,1113,84
1113 IF(FINK) 88,84,88
  84 IF(BC) 88,88,1111
```

```
 1111 IF(BRC) 88,88,89
   89 CALL BRCOR(R3,UU,XX,JK,VV,PM)
      MMM=N-1
      DO 90 J=1,3
      DO 90 K=1,3
   90 YY(J,K,MMM)=XX(J,K)
   88 GO TO 1001
   81 IF(NNBR(I)-1) 83,83,65
   83 CALL STATEB(SVIP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
   65 CALL FINBRA(U,VV,V,MAT,A,Z(I))
      DO 66 J=1,6
      DO 66 K=1,3
   66 VV(J,K)=V(J,K)
      IF(NNBR(I)) 68,67,67
   67 IF(NNBR(I)-3) 1001,68,68
   68 CALL BRANCH(R3,VV,PHI(II),U)
      N=N+1
      II=II+1
      MAT=0
      BC=SEVIM
      GO TO 63
 1001 CONTINUE
      GO TO 2000
C
C
C     CONTROL OUT OF PLANE
C
  998 P1=SEVIM
      P2=SEVIM
      D1=EEVIN
      D2=EEVIN
      Q1=SEVIM
      Q2=SEVIM
      Q3=SEVIM
      B1=EEVIN
      B2=EEVIN
      B3=EEVIN
      FINK=EEVIN
      IF(SVO(1)) 1013,1023,1013
 1013 CALL STAVEO(SVO(1),1,SVI)
      GO TO 1043
 1023 DO 1033 J=1,6
 1033 SVI(J,1)=SVOP(J,1)
 1043 IF(SVO(NMBR)) 1053,1063,1053
 1053 CALL STAVEO(SVO(NMBR),1,SVE)
      GO TO 1083
 1063 DO 1073 J=1,6
 1073 SVE(J,1)=SVOP(J,NMBR)
```

```
1083 JK=1
     IF(NBR) 1002,1002,1093
1093 DO 1095 N=2,NNN
     IF(SVO(N)) 1095,1095,1094
1094 CALL STAVEO(SVO(N),N,SVOP)
1095 CONTINUE
1002 N=2
     II=1
     HH=EEVIN
     DO 1003 I=1,NS
     BC=EEVIN
     MAT=0
     P(I)=CTX(I)+CLZ(I)+CTY(I)
     IF(P(I)) 1,2,1
   1 CALL HANGEO(CTX(I),CLZ(I),CTY(I),U)
     GO TO 14
   2 IF(AMYK) 3,11,3
   3 IF(RHO(I)) 4,8,4
   4 IF(Z(I)) 6,5,6
   5 CALL STIFOO(THETA(I),RHO(I),U)
     GO TO 14
   6 CALL CFIELO(AJT(I),R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),FF
    1AC,A)
   7 CALL POINO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)
     CALL MATMUL(A,B,U)
     MAT=1
     GO TO 14
   8 IF(Z(I)) 10, 9,10
   9 CALL RIGIO(AMU(I),TL(I),AIX(I),OMEGA(L),AIY(I),AJT(I),RI,U)
     GO TO 14
  10 CALL DISTMO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),AJT(I),R(I),D(I),
    1DI(I),G(I),SD,RI,FFAC,U)
     GO TO 14
  11 IF(RHO(I)) 4,12,4
  12 IF(Z(I)) 13,9,13
  13 CALL SFIELO(DI(I),TL(I),AJT(I),R(I),Z(I),G(I),SD,D(I),FFAC,A)
     GO TO 7
  14 IF(NNBR(I)) 201,200,201
 200 IF(I-1) 15,202,15
 202 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
     GO TO 15
  15 CALL FINMAT(U,UU,V,MAT,A,Z(I))
     DO 16 J=1,6
     DO 16 K=1,3
  16 UU(J,K)=V(J,K)
     IF(PM) 207,1114,85
1114 IF(FINK) 207,85,207
  85 IF(BC) 207,207,1112
```

```
1112 IF(BRC) 207,207,208
 208 CALL BRCOR(R3,UU,XX,JK,VV,PM)
     MMM=N-1
     DO 209 J=1,3
     DO 209 K=1,3
 209 YY(J,K,MMM)=XX(J,K)
 207 GO TO 1003
 201 IF(NNBR(I)-1) 204,204,17
 204 CALL STATEB(SVOP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
  17 CALL FINBRA(U,VV,V,MAT,A,Z(I))
     DO 18 J=1,6
     DO 18 K=1,3
  18 VV(J,K)=V(J,K)
     IF(NNBR(I)) 82,19,19
  19 IF(NNBR(I)-3) 1003,82,82
  82 CALL BRANCO(R3,VV,PHI(II),U)
     N=N+1
     II=II+1
     MAT=0
     BC=SEVIM
     GO TO 15
1003 CONTINUE
C
C    ITERATION
C
2000 CALL DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,PM,D1,
    1D2)
     IF(FINK) 87,87,86
  87 IF(PM) 99,99,98
  98 IF(KKK) 7030,7030,301
 301 GO TO 6001
7030 IF(FF-SEVIM) 7051,7060,7051
7060 KK=1
     M=M-1
     GO TO 3000
7051 IF(HH-SEVIM) 300,6001,300
 300 IF(SS) 303,302,302
 303 DEL(L)=X2-Y2
     D1=X1
     D2=(X2+Y2)/2.0D0
     GO TO 61
 302 DEL(L)=X1-Y1
     D1=(X1+Y1)/2.0D0
     D2=X2
     GO TO 61
  99 DEL(L)=V(1,1)*V(2,2)*V(3,3)-V(1,3)*V(2,2)*V(3,1)+V(1,2)*V(2,3)*
    1V(3,1)-V(1,2)*V(2,1)*V(3,3)+V(1,3)*V(3,2)*V(2,1)-V(1,1)*V(3,2)*
    2V(2,3)
```

```
      GO TO 61
   21 PDEL=DEL(L)
      GO TO 25
   22 IF(AA) 23,23,24
   23 PDEL=DEL(L-1)
      GO TO 25
   24 PDEL=DELC
   25 IF(PDEL*DEL(L)) 26,26,27
   26 IF(AA) 28,28,57
   27 BB=EEVIN
      IF(AA) 35,35,52
   28 AA=SEVIM
      BB=SEVIM
      DELC=DEL(L-1)
      OMEGAC=OMEGA(L-1)
      GO TO 59
   29 OMEGAM(M)=OMEGAC+(OMEGA(L)-OMEGAC)*DABS(DELC)/(DABS(DELC)+
     1DABS(DEL(L)))
      GO TO 32
   30 OMEGA(L+1)=OMEGA(L)-0.9D0*DOM
      DOM=0.5D0*DOM
   31 L=L+1
      BL=L
      IF(HL-BL) 7053,7053,86
 7053 KA=1
      GO TO 3000
   86 IF(JK) 1000,1000,1002
   32 BM=M
      IF(HM-BM) 3000,3000,33
   33 M=M+1
      IF(M-3) 340,34,34
   34 ADOM=(OMEGAM(M-1)-OMEGAM(M-2))*0.1D0
  340 AA=EEVIN
      BB=EEVIN
      CC=EEVIN
      GG=EEVIN
      DOM=ADOM
      DELC=EEVIN
      OMEGA(L+1)=OMEGAM(M-1)+2.0D0*CR
      OMEGAO=OMEGA(L+1)
      GO TO 31
   35 IF(OMEGA(L)-OMEGAG) 53,60,60
   50 OMEGA(L+1)=OMEGA(L)-(OMEGA(L)-OMEGAC)/2.0D0
      IF(OMEGA(L)-OMEGA(L+1)-CR) 51,51,31
   51 CC=SEVIM
      GO TO 31
   52 DELC=DEL(L)
      OMEGAC=OMEGA(L)
```

```
   53 OMEGA(L+1)=OMEGA(L)+DOM
      GO TO 31
   54 IF(DELC*DEL(L)) 29,29,55
   55 OMEGAM(M)=OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*DABS(DEL(L))/(DABS(DEL(L)
     1)+DABS(DEL(L-1)))
      GO TO 32
   56 IF(OMEGA(L)-OMEGAO) 21,21,22
   57 IF(BB) 58,58,50
   58 BB=SEVIM
   59 IF(OMEGA(L)-OMEGA(L-1)-CR) 29,29,30
   60 M=M-1
      GO TO 3000
   61 IF(GG) 70,70,72
   70 IF(DEL(L)) 77,71,77
   71 GG=SEVIM
      OMEGA(L+1)=OMEGA(L)-CR
      GO TO 31
   72 IF(GG-SEVIM) 73,73,74
   73 GG=2.0D0
      HDEL=DEL(L)
      OMEGA(L+1)=OMEGA(L)+2.0D0*CR
      GO TO 31
   74 TDEL=DEL(L)*HDEL
      IF(TDEL) 75,76,76
   75 OMEGAM(M)=OMEGA(L-2)
      GO TO 33
   76 KK=1
      M=M-1
      GO TO 3000
   77 IF(CC) 56,56,54
C
C     OUTPUT
C
 3000 IF(NM) 400,400,403
  400 WRITE(6,401)
  401 FORMAT(1H1//)
      WRITE(6,402)
  402 FORMAT (16HPROGRAM VIPIPE-1,34X,15H M.CETIN  DERIN,38X,14HDECEMBER
     1  1970//,10X,108H                    MODAL FREQUENCIES OF A PLANAR PIPING
     2 SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE /29H ME
     3THOD OF TRANSFER MATRICES.///119H ********************************
     4*****************************************************************
     5*******************      //)
  403 IF(NM)4058,4058,404
  404 WRITE(6,405)
  405 FORMAT(1H1//)
 4058 NM=1
 4059 IF(JK) 4060,4060,4065
```

```
4060 WRITE(6,4070) JJ
4070 FORMAT (50X,8HPROBLEM I2,//35X,47HIN PLANE MODE FREQUENCIES (RADIA
    1NS PER SECOND)   //45X,4HMODE,15X,10H FREQUENCY /)
     GO TO 4079
4065 WRITE(6,4075) JJ
4075 FORMAT(50X,8HPROBLEM I2,//33X,51HOUT OF PLANE MODE FREQUENCIES (RA
    1DIANS PER SECOND)      //45X,4HMODE,15X,10H FREQUENCY      /)
4079 MH=M
     WRITE(6,408) (M,OMEGAM(M),M=1,MH)
 408 FORMAT (46X,I2,17X,F25.16 /)
     IF(KA) 5000,5000,5010
5010 WRITE(6,5020)
5020 FORMAT(10X,53HPROBLEM TERMINATED DUE TO OMEGA STORAGE LIMITATION.
    1  /)
5000 IF(KK) 4080,4090,4080
4080 WRITE(6,4085)
4085 FORMAT(10X,69HPROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITAT
    1ION OF COMPUTER.      /)
4090 IF(AMYK)412,409,412
 409 WRITE(6,411)
 411 FORMAT (16X,53H             A LUMPED MASS APPROACH WAS EMPLOYED, A
    1ND )
     GO TO 414
 412 WRITE(6,413)
 413 FORMAT(24X,45HA DISTRIBUTED MASS APPROACH WAS EMPLOYED AND      )
 414 IF(SD)415,415,420
 415 IF(RI)416,416,418
 416 WRITE(6,417)
 417 FORMAT (2X,67HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA W
    1ERE CONSIDERED. /)
     GO TO 4610
 418 WRITE(6,419)
 419 FORMAT (3X,63HTHE EFFECTS OF SHEAR DEFLECTION WERE CONSIDERED WHIL
    1E THOSE OF     /3X,35HROTATIONAL INERTIA WERE NEGLECTED.         /)
     GO TO 4610
 420 IF(RI)421,421,423
 421 WRITE(6,422)
 422 FORMAT (3X,72HTHE EFFECTS OF ROTATIONAL INERTIA WERE CONSIDERED WH
    1ILE THOSE OF SHEAR           /3X,27HDEFLECTION WERE NEGLECTED.   /)
     GO TO 4610
 423 WRITE(6,424)
 424 FORMAT(3X,66HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WE
    1RE NEGLECTED. /)
4610 IF(PM) 425,425,4713
4713 IF(SS) 4611,4612,4612
4611 IF(PP-SEVIM) 4613,4614,4615
4613 WRITE(6,4623)
4623 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(A) USED.    /)
```

```
      GO TO 425
 4614 WRITE(6,4624)
 4624 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(B) USED.    /)
      GO TO 425
 4615 WRITE(6,4625)
 4625 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(C) USED.    /)
      GO TO 425
 4612 IF(SS-SEVIM) 4616,4617,4618
 4616 WRITE(6,4626)
 4626 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(D) USED.    /)
      GO TO 425
 4617 WRITE(6,4627)
 4627 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(E) USED.    /)
      GO TO 425
 4618 WRITE(6,4628)
 4628 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(F) USED.    /)
  425 IF(PM)9010,9020,9000
 9010 WRITE(6,9012)
 9012 FORMAT(24X,20HDELTA METHOD USED.   /)
      GO TO 9000
 9020 WRITE(6,9022)
 9022 FORMAT(24X,29HMODIFIED DELTA METHOD USED.    /)
 9000 WRITE(6,440)
  440 FORMAT(118H ******************************************************
     1**************************************************************    )
      IF(GDABC-SEVIM)4511,4520,4401
 4401 WRITE(6,4402)
 4402 FORMAT(35H SECTION LENGTH AND SUBSECTION DATA/23X,14HSECTION NUMBE
     1R,8X,25HLENGHT OF SECTION(INCHES),5X,21HNUMBER OF SECTIONS      )
      WRITE(6,443) (I,TL(I),Z(I),I=1,NS)
  443 FORMAT (29X,I3,23X,F7.2,21X,F4.0)
      LM=L
      IF(PM) 7054,7054,7055
 7055 WRITE(6,441)
  441 FORMAT (11H GRAPH DATA /22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
     1ADIANS PER SECOND),3X,30H   VALUE OF REMAINDER                  )
      GO TO 7057
 7054 WRITE(6,7056)
 7056 FORMAT (11H GRAPH DATA / 22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
     1ADIANS PER SECOND),3X,30HVALUE OF FREQUENCY DETERMINANT         )
 7057 WRITE(6,444) (L,OMEGA(L),DEL(L),L=1,LM)
  444 FORMAT (29X,I3,23X,F11.6,18X,D15.8)
 4520 WRITE(6,442)
  442 FORMAT(1X,10HINPUT DATA /)
      WRITE(6,4521)
 4521 FORMAT(1X,9HCOMPONENT,4X,8HDIAMETER,5X,14HWALL THICKNESS ,5X,6HLEN
     1GTH,5X,9HRADIUS OF,9X,8HINCLUDED,5X,7HDENSITY,5X,7HELASTIC,5X,5HSH
     2EAR /57X,9HCURVATURE,5X,12HANGLE OF ARC,16X,7HMODULUS,5X,7HMODULUS
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      3 /)
      DO 4525 I=1,NS
      IF(P(I))4524,4524,4523
 4524 WRITE(6,4522) I,D(I),DT(I),TL(I),RHO(I),THETR(I),BAMU(I),E(I),G(I)
 4522 FORMAT (3X,I3,8X,F7.3,10X,F6.3,7X,F8.3,4X,F8.3,9X,F6.2,8X,F7.2,4X,
     1E8.2,4X,E8.2)
      GO TO 4525
 4523 SH=SEVIM
 4525 CONTINUE
      IF(SH)395,395,4526
 4526 WRITE(6,4532)
 4532 FORMAT(//1X,7HHANGERS /)
      WRITE(6,4527)
 4527 FORMAT (2X,9HCOMPONENT,14X,3HCLX,14X,3HCLY,14X,3HCLZ,14X,3HCTX,14X
     1,3HCTY,14X,3HCTZ  /)
 4528 DO 4530 I=1,NS
      IF(P(I))4530,4530,4529
 4529 WRITE(6,4531) I,CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I),CTZ(I)
 4531 FORMAT(3X,I3,15X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.
     12 )
 4530 CONTINUE
  395 WRITE(6,448)
  448 FORMAT (//20H BOUNDARY CONDITIONS )
      WRITE(6,449)(SVI(I,1),I=1,6)
  449 FORMAT (5H SVI=6F3.0)
      WRITE(6,451) (SVE(I,1),I=1,6)
  451 FORMAT (5H SVE=6F3.0)
      IF(NBR) 4511,4511,4502
 4502 IF(JK)4505,4505,4515
 4505 WRITE(6,4510)((SVIP(I,J),I=1,6),J=2,NNN)
 4510 FORMAT (6F3.0)
      GO TO 4511
 4515 WRITE(6,4517)((SVOP(I,J),I=1,6),J=2,NNN)
 4517 FORMAT(6F3.0)
 4511 CONTINUE
C     SORT
      IF(GRAPH-SEVIM) 6001,6000,6001
 6000 LM1=LM-1
      DO 6100 L=1,LM
      EDL(L)=DEL(L)
 6100 CONTINUE
      DO 6600 L=1,LM1
      LP1=L+1
      DO 6600 M=LP1,LM
      IF(OMEGA(L)-OMEGA(M)) 6600,6600,6003
 6003 TEMP=OMEGA(L)
      OMEGA(L)=OMEGA(M)
      OMEGA(M)=TEMP
```

```
      TEMP=EDL(L)                                                       006190
      EDL(L)=EDL(M)                                                     006200
      EDL(M)=TEMP                                                       006210
 6600 CONTINUE                                                          006220
C     GRAPH                                                             006230
      NUMPTS=60                                                         006240
      LT1=LM/NUMPTS                                                     006250
      IF(LT1) 6024,6025,6024                                            006255
 6025 NUMPTS=LM                                                         006270
      L=1                                                               006285
      DO 2015 J=1,60                                                    006300
      ZERGU(J)=OMEGA(J)                                                 006315
      CET(J)=EDL(J)                                                     006330
 2015 CONTINUE                                                          006345
      CALL DRAW(NUMPTS,ZERGU,CET,0,0,LABEL,ITITLE,0,0,0,0,0,0,9,10,1,   006360
     1LAST)                                                             006375
      GO TO 6001                                                        006390
 6024 DO 6020 J=1,17                                                    006400
      LT2=0                                                             006410
      LT2=LT2+J*NUMPTS                                                  006420
      IF(LT2-LM)6020,6021,6022                                          006430
 6020 CONTINUE                                                          006440
 6021 LT=LT1                                                            006450
      GO TO 6023                                                        006460
 6022 LT=LT1+1                                                          006470
      NUMEND=LM-LT1*NUMPTS                                              006480
 6023 L=1                                                               006490
      DO 7021 I=1,LT                                                    006500
      NEVIN1=I-3                                                        006505
      NEVIN2=I-5                                                        006510
      NEVIN3=I-7                                                        006515
      NEVIN4=I-9                                                        006520
      NEVIN5=I-LT                                                       006525
      IF(I-1) 6008,6009,6008                                            006530
 6008 IF(I-LT) 8011,6011,6011                                           006535
 8011 IF(I-3) 6010,6010,8012                                            006540
 8012 IF(I-5) 6010,6010,8013                                            006545
 8013 IF(I-7) 6010,6010,8014                                            006550
 8014 IF(I-9) 6010,6010,8015                                            006555
 8015 IF(I-LT) 6010,6011,6011                                           006560
 6009 DO 6031 N=1,NUMPTS                                                006780
      EDL(N)=EDL(L+N-1)                                                 006790
      CET(N)=EDL(N)                                                     006795
      OMEGA(N)=OMEGA(L+N-1)                                             006800
 6031 ZERGU(N)=OMEGA(N)                                                 006805
      WRITE(6,6027) NUMPTS,EDL(1)                                       006810
 6027 FORMAT(1H1,//////5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)              006820
      CALL DRAW(NUMPTS,ZERGU,CET,1,0,LABEL1,ITITLE,0,0,0,0,0,0,9,10,1,  006830
```

```
     1LAST)
      GO TO 7021
 6010 L=L+NUMPTS
      OMEGAX=OMEGA(L)
      DO 6030 N=1,NUMPTS
      EDL(N)=EDL(N+L-1)
      CET(N)=EDL(N)
      OMEGA(N)=OMEGA(L+N-1)
      OMEGA(N)=OMEGA(N)-OMEGAX
 6030 ZERGU(N)=OMEGA(N)
      WRITE(6,6032) NUMPTS,EDL(1)
 6032 FORMAT(///////5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)
      IF(NEVIN1.LT.0) GO TO 2002
      IF(NEVIN1.EQ.0) GO TO 2003
      IF(NEVIN2.LT.0) GO TO 2004
      IF(NEVIN2.EQ.0) GO TO 2005
      IF(NEVIN3.LT.0) GO TO 2006
      IF(NEVIN3.EQ.0) GO TO 2007
      IF(NEVIN4.LT.0) GO TO 2008
      IF(NEVIN4.EQ.0) GO TO 2009
      IF(NEVIN5.LT.0) GO TO 2010
 2002 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL2,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2003 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL3,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2004 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL4,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2005 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL5,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2006 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL6,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2007 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL7,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2008 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL8,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2009 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL9,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
 2010 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL0,ITITLE,0,0,0,0,0,0,9,10,1,
     1LAST)
      GO TO 7021
```

```
6011 L=L+NUMPTS
     OMEGAX=OMEGA(L)
     DO 6033 N=1,NUMEND
     EDL(N)=EDL(N+L-1)
     CET(N)=EDL(N)
     OMEGA(N)=OMEGA(N+L-1)
     OMEGA(N)=OMEGA(N)-OMEGAX
6033 ZERGU(N)=OMEGA(N)
     WRITE(6,6051) NUMEND,EDL(1)
6051 FORMAT(///////5X,7HNUMEND=,I3,5X,7HEDL(1)=,E12.9)
     CALL DRAW(NUMEND,ZERGU,CET,3,0,LABELL,ITITLE,0,0,0,0,0,0,9,10,1,
    1LAST)
     GO TO 6001
7021 CONTINUE
6001 IF(HH-SEVIM) 8024,8023,8023
8023 MH=M
     LM=L
8024 DO 446 M=1,MH
 446 OMEGAM(M)=EEVIN
     DO 447 L=1,LM
     OMEGA(L)=EEVIN
     EDL(L)=EEVIN
 447 DEL(L)=EEVIN
     DELC=EEVIN
     OMEGAC=EEVIN
     IF(HH-SEVIM) 8001,8022,8001
8001 IF(PM) 8031,8031,8033
8033 IF(REM)8031,8032,8032
8031 IF(MN)8022,8022,452
8032 IF(REM-SEVIM) 8041,8042,8043
8041 IF(PP-SEVIM) 8043,8043,8072
8072 IF(SS) 8073,8074,8074
8073 SS=EEVIN
     GO TO 8022
8074 IF(SS-SEVIM) 8042,8042,8075
8075 SS=-SEVIM
     PP=EEVIN
     REM=-SEVIM
     GO TO 8033
8042 SS=SS-SEVIM
     IF(SS)8051,8052,8053
8051 SS=SEVIM
     GO TO 8022
8052 SS=2.0D0
     GO TO 8022
8053 SS=-SEVIM
     REM=-SEVIM
     GO TO 8033
```

```
8043 PP=PP-SEVIM
     IF(PP) 8061,8062,8063
8061 PP=SEVIM
     GO TO 8022
8062 PP=2.0D0
     GO TO 8022
8063 PP=EEVIN
     REM=-SEVIM
     GO TO 8033
 452 PP=EEVIN
     RR=EEVIN
     SS=-SEVIM
8022 L=1
     M=1
     AA=EEVIN
     BB=EEVIN
     CC=EEVIN
     FF=EEVIN
     GG=EEVIN
     KK=0
     KA=0
     KKK=0
     ADOM=BDOM
     CR=BR
     OMEGA(1)=OMEGAI
     OMEGAO=OMEGAI
     DOM=ADOM
     IF(HH-SEVIM) 8021,7040,8021
7040 IF(SS) 8191,8192,8192
8191 IF(PP-SEVIM) 8091,8092,8093
8192 IF(SS-SEVIM) 8291,8292,8293
8091 WRITE(6,8081)
8081 FORMAT(1H1,////18HMETHOD  A  FAILED.  ///)
     GO TO 8021
8092 WRITE(6,8082)
8082 FORMAT(1H1,////18HMETHOD  B  FAILED. ///)
     GO TO 8021
8093 WRITE(6,8084)
8084 FORMAT(1H1,////18HMETHOD  C  FAILED. ///)
     GO TO 8021
8291 WRITE(6,8281)
8281 FORMAT(1H1,////18HMETHOD  D  FAILED. ///)
     GO TO 8021
8292 WRITE(6,8282)
8282 FORMAT(1H1,////18HMETHOD  E  FAILED. ///)
     GO TO 8021
8293 WRITE(6,8283)
8283 FORMAT(1H1,////18HMETHOD  F  FAILED. ///)
```

```
8021 IF(PM) 453,453,8035
8035 IF(REM) 8037,8036,8036
8036 IF(JK) 998,999,998
8037 REM=ERM
 453 IF(PM) 6081,6082,6080
6081 IF(PMM-SEVIM) 6084,6080,6084
6084 IF(PMM*0.1D0-10.1D0) 6086,6085,6086
6085 PM=SEVIM
     GO TO 8036
6086 PM=EEVIN
     GO TO 8036
6082 IF(PMM*0.1D0-1.1D0) 6080,6080,6085
6080 IF(MN) 9001,9001,8034
8034 MN=0
     PM=PPM
     GO TO 998
9001 JJ=JJ+1
     DO 454 I=1,NS
     P(I)=EEVIN
 454 Z(I)=EEVIN
     GO TO 450
 600 STOP
     END


     SUBROUTINE SUBSEC (RHO,THETA,D,TL,HM,Z,OMEGAI,HMH)
     IMPLICIT REAL*8(A-H,O-Z)
     RHOD=DABS(RHO)
     IF(HMH) 41,40,41
  40 IF(HM-5.0D0) 8,8,7
   8 IF(OMEGAI-800.0D0) 10,7,7
   7 HM=6.0D0
     GO TO 10
  41 HM=HMH
  10 RA=TL/D
     IF(RHOD-0.0D0) 3,3,4
   3 IF(RA-1.0D0) 5,5,6
   5 Z=0.0D0
     RETURN
   6 IF(RA-3.0D0) 15,15,16
  15 Z=2.0D0
     RETURN
  16 IF(RA-6.0D0) 17,17,18
  17 Z=3.0D0
     RETURN
  18 IF(HM-3.0D0) 1,1,2
   1 Z=6.0D0
     RETURN
```

```
    2 Z=HM*2.0D0
      RETURN
    4 RD=RHOD/D
      IF(RD-1.0D0) 19,19,23
   19 Z=0.0D0
      RETURN
   23 IF(RD-3.0D0) 24,24,25
   24 IF(THETA-0.44D0) 26,26,27
   26 Z=0.0D0
      RETURN
   27 IF(THETA-1.57D0) 28,28,29
   28 Z=4.0D0
      RETURN
   29 Z=6.0D0
      RETURN
   25 IF(RD-6.0D0) 30,30,31
   30 IF(THETA-0.44D0) 32,32,33
   32 Z=4.0D0
      RETURN
   33 IF(THETA-1.57D0) 34,34,35
   34 Z=6.0D0
      RETURN
   35 Z=8.0D0
      RETURN
   31 IF(THETA-0.44D0) 36,36,3
   36 Z=4.0D0
      RETURN
      END

      SUBROUTINE DISTM (AMU,AIY,OMEGA,TL,R,D,DI,G,SD,RI,E,FFAC,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      ARA=(3.141592654D0/4.0D0)*(D**2-DI**2)
      AREA=ARA*FFAC
      BE=TL*OMEGA*DSQRT(AMU/(E*ARA))
      P4=R*AMU*TL**4*OMEGA**2
      IF(RI) 1,1,2
    1 T=R*AMU*(AIY*OMEGA*TL)**2
      GO TO 3
    2 T=0.0D0
    3 IF(SD) 4,4,5
    4 S=(AMU*(OMEGA*TL)**2)/(G*AREA)
      GO TO 6
    5 S=0.0D0
    6 SPT=S+T
      VV=DSQRT(P4+(S-T)**2/4.0D0)
      AL1=DSQRT(VV-SPT*0.5D0)
```

```
      AL2=DSQRT(VV+SPT*0.5D0)
      CL=1.0D0/(AL1**2+AL2**2)
      CHL1=(DEXP(AL1)+1.0D0/DEXP(AL1))/2.0D0
      SHL1=(DEXP(AL1)-1.0D0/DEXP(AL1))/2.0D0
      CL2=DCOS(AL2)
      SL2=DSIN(AL2)
      SBE=DSIN(BE)
      CBE=DCOS(BE)
      C00=CL*(AL2**2*CHL1+AL1**2*CL2)
      C01=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)
      C02=CL*(CHL1-CL2)
      C03=CL*(SHL1/AL1-SL2/AL2)
      DO 7 J=1,6
      DO 7 K=1,6
    7 U(J,K)=0.0D0
      U(1,1)=CBE
      U(1,6)=TL*SBE/(BE*ARA*E)
      U(2,2)=C00-S*C02
      U(2,3)=TL*(C01-SPT*C03)
      U(2,4)=R*C02*TL**2
      U(2,5)=(R*TL**3/P4)*((P4+S**2)*C03-S*C01)
      U(3,2)=P4*C03/TL
      U(3,3)=C00-T*C02
      U(3,4)=TL*R*(C01-T*C03)
      U(3,5)=U(2,4)
      RR=1.0D0/R
      U(4,2)=P4*RR*C02/TL**2
      U(4,3)=RR*((P4+T**2)*C03-T*C01)/TL
      U(4,4)=U(3,3)
      U(4,5)=U(2,3)
      U(5,2)=P4*RR*(C01-S*C03)/TL**3
      U(5,3)=U(4,2)
      U(5,4)=U(3,2)
      U(5,5)=U(2,2)
      U(6,1)=-AMU*TL*OMEGA**2*SBE/BE
      U(6,6)=CBE
      RETURN
      END

      SUBROUTINE DISTMO (AMU,AIX,AIY,OMEGA,TL,AJT,R,D,DI,G,SD,RI,FFAC,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      W=1.0D0/(G*AJT)
      AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
      BE=+DSQRT(AMU*(TL*AIX*OMEGA)**2*W)
      P4=R*AMU*TL**4*OMEGA**2
      IF(RI) 2,2,3
```

```
    2 T=R*AMU*(AIY*OMEGA*TL)**2                                    009330
      GO TO 4                                                      009340
    3 T=0.0D0                                                      009350
    4 IF(SD) 5,5,6                                                 009360
    5 S=(AMU*(OMEGA*TL)**2)/(G*AREA)                               009370
      GO TO 7                                                      009380
    6 S=0.0D0                                                      009390
    7 VV=+DSQRT(P4+(S-T)**2/4.0D0)                                 009400
      SPT=S+T                                                      009410
      AL1=+DSQRT(VV-SPT*0.5D0)                                     009420
      AL2=+DSQRT(VV+SPT*0.5D0)                                     009430
      CL=1.0D0/(AL1**2+AL2**2)                                     009440
      CHL1=(DEXP(AL1)+1.0D0/DEXP(AL1))/2.0D0                       009450
      SHL1=(DEXP(AL1)-1.0D0/DEXP(AL1))/2.0D0                       009460
      CL2=DCOS(AL2)                                                009470
      SL2=DSIN(AL2)                                                009480
      SBE=DSIN(BE)                                                 009490
      CBE=DCOS(BE)                                                 009500
      C00=CL*(AL2**2*CHL1+AL1**2*CL2)                              009510
      C01=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)                  009520
      C02=CL*(CHL1-CL2)                                            009530
      C03=CL*(SHL1/AL1-SL2/AL2)                                    009540
      DO 1 J=1,6                                                   009550
      DO 1 K=1,6                                                   009560
    1 U(J,K)=0.0D0                                                 009570
      U(1,1)=CBE                                                   009580
      U(1,2)=TL*W*SBE/BE                                           009590
      U(2,1)=-AMU*TL*AIX**2*SBE/BE                                 009600
      U(2,2)=CBE                                                   009610
      U(3,3)=C00-S*C02                                             009620
      U(3,4)=TL*(C01-SPT*C03)                                      009630
      U(3,5)=R*C02*TL**2                                           009633
      U(3,6)=(R*TL**3/P4)*(-S*C01+(P4+S**2)*C03)                   009634
      U(4,3)=P4*C03/TL                                             009635
      U(4,4)=C00-T*C02                                             009636
      U(4,5)=TL*R*(C01-T*C03)                                      009637
      U(4,6)=U(3,5)                                                009638
      RR=1.0D0/R                                                   009639
      U(5,3)=P4*RR*C02/TL**2                                       009640
      U(5,4)=RR*(-T*C01+(P4+T**2)*C03)/TL                          009641
      U(5,5)=U(4,4)                                                009642
      U(5,6)=U(3,4)                                                009643
      U(6,3)=P4*RR*(C01-S*C03)/TL**3                               009644
      U(6,4)=U(5,3)                                                009645
      U(6,5)=U(4,3)                                                009646
      U(6,6)=U(3,3)                                                009647
      RETURN                                                       009648
      END                                                          009649
```

```
      SUBROUTINE RIGID (AMU,TL,OMEGA,AIY,RI,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      AM=AMU*TL
      DO 1 J=1,6
      DO 1 K=1,6
    1 U(J,K)=0.0D0
      U(1,1)=1.0D0
      U(6,1)=-AM*OMEGA**2
      U(6,6)=1.0D0
      U(2,2)=1.0D0
      U(2,3)=TL
      U(3,3)=1.0D0
      U(4,2)=AM*TL*OMEGA**2/2.0D0
      IF(RI) 2,2,3
    2 U(4,3)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
      GO TO 4
    3 U(4,3)=AM*(OMEGA*TL)**2/6.0D0
    4 U(4,4)=1.0D0
      U(4,5)=TL
      U(5,2)=-U(6,1)
      U(5,3)=U(4,2)
      U(5,5)=1.0D0
      RETURN
      END

      SUBROUTINE RIGIO (AMU,TL,AIX,OMEGA,AIY,AJT,SDRI,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      DO 2 J=1,6
      DO 2 K=1,6
    2 U(J,K)=0.0D0
      AM=AMU*TL
      U(1,1)=1.0D0
      U(2,1)=-AMU*TL*(AIX*OMEGA)**2
      U(2,2)=1.0D0
      U(3,3)=1.0D0
      U(3,4)=TL
      U(4,4)=1.0D0
      U(5,3)=AM*TL*OMEGA**2/2.0D0
      IF(SDRI-0.0D0) 30,30,31
   30 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
      GO TO 1
   31 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0)
    1 U(5,5)=1.0D0
```

```
      U(5,6)=TL
      U(6,3)=AM*OMEGA**2
      U(6,4)=U(5,3)
      U(6,6)=1.0D0
      RETURN
      END

      SUBROUTINE STIFCO (THETA,RHO,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      DO 1 J=1,6
      DO 1 K=1,6
    1 U(J,K)=0.0D0
      IF(RHO) 2,2,3
    2 V=-1.0D0
      GO TO 4
    3 V=1.0D0
    4 CT=DCOS(THETA)
      ST=DSIN(THETA)
      U(1,1)=CT
      U(1,2)=-V*ST
      U(2,1)=V*ST
      U(2,2)=CT
      U(3,3)=1.0D0
      U(4,4)=1.0D0
      U(5,5)=CT
      U(5,6)=-V*ST
      U(6,5)=V*ST
      U(6,6)=CT
      RETURN
      END

      SUBROUTINE STIFOO (THETA,RHO,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6)
      IF(RHO) 1,1,2
    1 V=-1.0D0
      GO TO 3
    2 V=1.0D0
    3 CT=DCOS(THETA)
      ST=DSIN(THETA)
      DO 4 J=1,6
      DO 4 K=1,6
    4 U(J,K)=0.0D0
      U(1,1)=CT
      U(1,4)=ST*V
```

```
      U(2,2)=CT
      U(2,5)=ST*V
      U(3,3)=1.0D0
      U(4,1)=-ST*V
      U(4,4)=CT
      U(5,2)=-ST*V
      U(5,5)=CT
      U(6,6)=1.0D0
      RETURN
      END

      SUBROUTINE STAVEC(SV,N,BC)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BC(6,22)
      DO 10 J=1,6
   10 BC(J,N)=0.0D0
      IF(SV-1.0D0) 1,1,2
    1 BC(4,N)=1.0D0
      BC(5,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
    2 IF(SV-2.0D0) 3,3,4
    3 BC(1,N)=1.0D0
      BC(2,N)=1.0D0
      BC(3,N)=1.0D0
      RETURN
    4 IF(SV-3.0D0) 5,5,6
    5 BC(3,N)=1.0D0
      BC(5,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
    6 IF(SV-4.0D0) 7,7,8
    7 BC(1,N)=1.0D0
      BC(3,N)=1.0D0
      BC(5,N)=1.0D0
      RETURN
    8 BC(2,N)=1.0D0
      BC(3,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
      END
```

```
      SUBROUTINE STAVEO (SV,N,BC)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BC(6,22)
      DO 10 L=1,6
   10 BC(L,N)=0.0D0
      IF(SV-1.0D0) 1,1,2
    1 BC(2,N)=1.0D0
      BC(5,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
    2 IF(SV-2.0D0) 3,3,4
    3 BC(1,N)=1.0D0
      BC(3,N)=1.0D0
      BC(4,N)=1.0D0
      RETURN
    4 IF(SV-3.0D0) 5,5,6
    5 BC(2,N)=1.0D0
      BC(4,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
    6 IF(SV-4.0D0) 7,7,5
    7 BC(1,N)=1.0D0
      BC(4,N)=1.0D0
      BC(6,N)=1.0D0
      RETURN
      END


      SUBROUTINE INVERT (A,N,D,L,M)
      IMPLICIT REAL*8(A-H,O-Z)
C     PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX
      DIMENSION A(25,25),L(25),M(25)
C     SEARCH FOR LARGEST ELEMENT
      D=1.0D0
      DO 80 K=1,N
      L(K)=K
      M(K)=K
      BIGA=A(K,K)
      DO 20 I=K,N
      DO 20 J=K,N
      IF(DABS(BIGA)-DABS(A(I,J))) 10,20,20
   10 BIGA=A(I,J)
      L(K)=I
      M(K)=J
   20 CONTINUE
C     INTERCHANGE ROWS
      J=L(K)
      IF(L(K)-K) 35,35,25
```

```
   25 DO 30 I=1,N
      HOLD=-A(K,I)
      A(K,I)=A(J,I)
   30 A(J,I)=HOLD
C     INTERCHANGE COLUMNS
   35 I=M(K)
      IF(M(K)-K) 45,45,37
   37 DO 40 J=1,N
      HOLD=-A(J,K)
      A(J,K)=A(J,I)
   40 A(J,I)=HOLD
C     DIVIDE COLUMN BY MINUS PIVOT
   45 DO 55 I=1,N
   46 IF(I-K) 50,55,50
   50 A(I,K)=A(I,K)/(-A(K,K))
   55 CONTINUE
C     REDUCE MATRIX
      DO 65 I=1,N
      DO 65 J=1,N
   56 IF(I-K) 57,65,57
   57 IF(J-K) 60,65,60
   60 A(I,J)=A(I,K)*A(K,J)+A(I,J)
   65 CONTINUE
C     DIVIDE ROW BY PIVOT
      DO 75 J=1,N
   68 IF(J-K) 70,75,70
   70 A(K,J)=A(K,J)/A(K,K)
   75 CONTINUE
C     CONTINUED PRODUCT OF PIVOTS
      D=D*A(K,K)
C     REPLACE PIVOT BY RECIPROCAL
      A(K,K)=1.0D0/A(K,K)
   80 CONTINUE
C     FINAL ROW AND COLUMN INTERCHANGE
      K=N
  100 K=K-1
      IF(K) 150,150,103
  103 I=L(K)
      IF(I-K) 120,120,105
  105 DO 110 J=1,N
      HOLD=A(J,K)
      A(J,K)=-A(J,I)
  110 A(J,I)=HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 DO 130 I=1,N
      HOLD=A(K,I)
      A(K,I)=-A(J,I)
```

```
130 A(J,I)=HOLD
    GO TO 100
150 RETURN
    END

    SUBROUTINE HANGER (CLX,CLY,CTZ,U)
    IMPLICIT REAL*8(A-H,O-Z)
    DIMENSION U(6,6)
    ZZ=1.0D0
    DO 1 J=1,6
    DO 1 K=1,6
  1 U(J,K)=0.0D0
    U(1,1)=ZZ
    U(2,2)=ZZ
    U(3,3)=ZZ
    U(4,4)=ZZ
    U(5,5)=ZZ
    U(6,6)=ZZ
    U(4,3)=CTZ
    U(5,2)=-CLY
    U(6,1)=CLX
    RETURN
    END

    SUBROUTINE HANGEO (CTX,CLZ,CTY,U)
    IMPLICIT REAL*8(A-H,O-Z)
    DIMENSION U(6,6)
    ZZ=1.0D0
    DO 1 J=1,6
    DO 1 K=1,6
  1 U(J,K)=0.0D0
    U(1,1)=ZZ
    U(2,1)=CTX
    U(2,2)=ZZ
    U(3,3)=ZZ
    U(4,4)=ZZ
    U(5,4)=CTY
    U(5,5)=ZZ
    U(6,3)=-CLZ
    U(6,6)=ZZ
    RETURN
    END
```

```
      SUBROUTINE CFIELO(AJT,R,Z,G,SDRI,RHO,THETA,D,DI,FFAC,A)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      PHI=THETA/Z
      RHOD=RHO
      IF(RHO-ZA) 11,11,12
   11 V=-ZZ
      GO TO 1
   12 V=ZZ
    1 CP=DCOS(PHI)
      SP=DSIN(PHI)
      W=ZZ/(G*AJT)
      F1=(PHI*CP+SP)/2.0D0
      F3=(SP-CP*PHI)/2.0D0
      F5=(2.0D0-2.0D0*CP-PHI*SP)/2.0D0
      F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)/2.0D0
      RHO=DABS(RHO)
      DO 2 J=1,6
      DO 2 K=1,6
    2 A(J,K)=ZA
      IF(SDRI) 13,13,14
   13 AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
      SD=RHO*THETA/(G*Z*AREA)
      GO TO 15
   14 SD=ZA
   15 A(1,1)=CP
      A(1,2)=(W*RHO*F1)-(RHO*F3*R)
      A(1,4)=V*SP
      A(1,5)=V*(W+R)*(RHO*PHI*SP)/2.0D0
      A(1,6)=V*(W+R)*(RHO**2)*F3
      A(2,2)=CP
      A(2,5)=V*SP
      A(2,6)=V*RHO*(ZZ-CP)
      A(3,1)=-A(2,6)
      A(3,2)=-ZZ*A(1,6)
      A(3,3)=ZZ
      A(3,4)=RHO*SP
      A(3,5)=((R*(RHO**2)*PHI*SP)/2.0D0)-W*RHO**2*F5
      A(3,6)=(R*RHO**3*F3)-(W*F6*RHO**3)-SD
      A(4,1)=-V*SP
      A(4,2)=-V*(W+R)*RHO*PHI*SP/2.0D0
      A(4,4)=CP
      A(4,5)=(R*RHO*F1)-(W*RHO*F3)
      A(4,6)=A(3,5)
      A(5,2)=-V*SP
      A(5,5)=CP
```

```
      A(5,6)=A(3,4)
      A(6,6)=ZZ
      RHO=RHOD
      RETURN
      END

      SUBROUTINE POINO(AMU,AIX,AIY,OMEGA,TL,Z,SDRI,B)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      AM=AMU*TL/(Z-ZZ)
      DO 1 J=1,6
      DO 1 K=1,6
    1 B(J,K)=ZA
      B(1,1)=ZZ
      B(2,1)=-AM*(AIX*OMEGA)**2
      B(2,2)=ZZ
      B(3,3)=ZZ
      B(4,4)=ZZ
      IF(SDRI-ZA) 9,9,10
    9 B(5,4)=-AM*(AIY*OMEGA)**2
      GO TO 11
   10 B(5,4)=ZA
   11 B(5,5)=ZZ
      B(6,3)=AM*OMEGA**2
      B(6,6)=ZZ
      RETURN
      END

      SUBROUTINE CFIELD(R,Z,G,SD,RHO,THETA,D,DI,E,A)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      PHI=THETA/Z
      RD=DABS(RHO)
      IF(RHO)1,1,2
    1 V=-ZZ
      GO TO 3
    2 V=ZZ
    3 CP=DCOS(PHI)
      SP=DSIN(PHI)
      DO 4 J=1,6
      DO 4 K=1,6
    4 A(J,K)=ZA
```

```
      A(1,1)=CP
      A(1,2)=-V*SP
      A(1,3)=-V*RD*(ZZ-CP)
      A(1,4)=-V*RD**2*R*(PHI-SP)
      AR=3.141592654D0*(D**2-DI**2)/4.0D0
      W=ZZ/(E*AR)
      F3=(SP-PHI*CP)*0.50D0
      F1= (PHI*CP+SP)*0.50D0
      F5=(2.0D0-2.0D0*CP-PHI*SP)*0.50D0
      F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)*0.50D0
      A(1,5)=V*(RD*W*PHI*SP*0.5D0-R*F5*RD**3)
      A(1,6)=RD*W*F1+R*F6*RD**3
      A(2,1)=V*SP
      A(2,2)=CP
      A(2,3)=RD*SP
      A(2,4)=R*RD**2*(ZZ-CP)
      A(2,5)=RD*F3*(W+R*RD**2)
      A(2,6)=A(1,5)
      A(3,3)=ZZ
      A(3,4)=RD*R*PHI
      A(3,5)=A(2,4)
      A(3,6)=A(1,4)
      A(4,4)=ZZ
      A(4,5)=A(2,3)
      A(4,6)=A(1,3)
      A(5,5)=CP
      A(5,6)=A(1,2)
      A(6,5)=A(2,1)
      A(6,6)=CP
      RETURN
      END

      SUBROUTINE POINT(AMU,AIY,OMEGA,TL,Z,RI,B)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      AM=AMU*TL/(Z-ZZ)
      DO 1 J=1,6
      DO 1 K=1,6
    1 B(J,K)=ZA
      B(1,1)=ZZ
      B(2,2)=ZZ
      B(3,3)=ZZ
      IF(RI)2,2,3
    2 B(4,3)=-AM*(AIY*OMEGA)**2
    3 B(4,4)=ZZ
```

```
      B(5,2)=AM*OMEGA**2
      B(5,5)=ZZ
      B(6,1)=-B(5,2)
      B(6,6)=ZZ
      RETURN
      END

      SUBROUTINE SFIELD(DI,TL,R,Z,G,SD,D,E,FFAC,A)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      ARA=(3.141592654D0/4.0D0)*(D**2-DI**2)
      AREA=ARA*FFAC
      DL=TL/Z
      DO 1 J=2,5
    1 A(1,J)=ZA
      A(1,1)=ZZ
      A(1,6)=DL/(E*ARA)
      A(2,1)=ZA
      A(2,2)=ZZ
      A(2,3)=DL
      A(2,4)=DL**2*R/2.0D0
      IF(SD) 2,2,3
    2 A(2,5)=DL**3*R/6.0D0-DL/(G*AREA)
      GO TO 4
    3 A(2,5)=DL**3*R/6.0D0
    4 A(2,6)=ZA
      A(3,1)=ZA
      A(3,2)=ZA
      A(3,3)=ZZ
      A(3,4)=DL*R
      A(3,5)=A(2,4)
      A(3,6)=ZA
      DO 5  J=1,6
      A(4,J)=ZA
      A(5,J)=ZA
    5 A(6,J)=ZA
      A(4,4)=ZZ
      A(4,5)=DL
      A(5,5)=ZZ
      A(6,6)=ZZ
      RETURN
      END
```

```
      SUBROUTINE SFIELO(DI,TL,AJT,R,Z,G,SDRI,D,FFAC,A)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6)
      ZZ=1.0D0
      ZA=0.0D0
      AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
      DL=TL/Z
      DO 2 J=1,6
      DO 2 K=1,6
    2 A(J,K)=ZA
      A(1,1)=ZZ
      A(2,2)=ZZ
      A(3,3)=ZZ
      A(4,4)=ZZ
      A(5,5)=ZZ
      A(6,6)=ZZ
      IF(SDRI-ZA) 7,7,8
    7 A(3,6)=DL**3*R/6.0D0-DL/(G*AREA)
      GO TO 1
    8 A(3,6)=DL**3*R/6.0D0
    1 A(1,2)=DL/(G*AJT)
      A(3,4)=DL
      A(3,5)=DL**2*R/2.0D0
      A(4,5)=DL*R
      A(4,6)=A(3,5)
      A(5,6)=DL
      RETURN
      END

      SUBROUTINE MATMUL(A,B,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(6,6),B(6,6),C(6,6),U(6,6)
      DO 1 J=1,6
      DO 1 K=1,6
      C(J,K)=0.0D0
      DO 1 L=1,6
    1 C(J,K)=C(J,K)+B(J,L)*A(L,K)
      DO 2 J=1,6
      DO 2 K=1,6
    2 U(J,K)=C(J,K)
      RETURN
      END
```

```
      SUBROUTINE FINBRA(U,VV,V,MAT,A,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6),A(6,6),VV(6,3),V(6,3)
      IF(MAT) 65,64,65
   64 DO 67 J=1,6
      DO 67 K=1,3
      V(J,K)=0.0D0
      DO 67 L=1,6
   67 V(J,K)=V(J,K)+U(J,L)*VV(L,K)
      RETURN
   65 N=Z-1.0D0
      DO 5 M=1,N
      DO 4 J=1,6
      DO 4 K=1,3
      V(J,K)=0.0D0
      DO 4 L=1,6
    4 V(J,K)=V(J,K)+U(J,L)*VV(L,K)
      DO 5  J=1,6
      DO 5  K=1,3
    5 VV(J,K)=V(J,K)
      DO 7 J=1,6
      DO 7 K=1,3
      V(J,K)=0.0D0
      DO 7 L=1,6
    7 V(J,K)=V(J,K)+A(J,L)*VV(L,K)
      RETURN
      END

      SUBROUTINE FINMAT(U,UU,V,MAT,A,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(6,6),A(6,6),V(6,3),UU(6,3)
      ZA=0.0D0
      IF(MAT) 65,64,65
   64 DO 67 J=1,6
      DO 67 K=1,3
      V(J,K)=ZA
      DO 67 L=1,6
   67 V(J,K)=V(J,K)+U(J,L)*UU(L,K)
      RETURN
   65 N=Z-1.0D0
      DO 5 M=1,N
      DO 4 J=1,6
      DO 4 K=1,3
      V(J,K)=ZA
      DO 4 L=1,6
    4 V(J,K)=V(J,K)+U(J,L)*UU(L,K)
      DO 5 J=1,6
```

```
      DO 5 K=1,3
    5 UU(J,K)=V(J,K)
      DC 7 J=1,6
      DO 7 K=1,3
      V(J,K)=ZA
      DO 7 L=1,6
    7 V(J,K)=V(J,K)+A(J,L)*UU(L,K)
      RETURN
      END

      SUBROUTINE BRANCH(R3,VV,PHI,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R3(3,3),VV(6,3),U(6,6),R1(25,25),LL(25),MM(25),R2(3,3),
     1R(3,3),G1(3,3),G2(3,3)
      ZZ=1.0D0
      ZA=0.0D0
      DO 4 L=1,3
      R1(1,L)=VV(1,L)
      R1(2,L)=VV(2,L)
      R1(3,L)=VV(3,L)
      R2(1,L)=VV(4,L)
      R2(2,L)=VV(5,L)
    4 R2(3,L)=VV(6,L)
      SP=DSIN(PHI)
      CP=DCOS(PHI)
      G1(1,1)=CP
      G1(1,2)=-SP
      G1(1,3)=ZA
      G1(2,1)=SP
      G1(2,2)=CP
      G1(2,3)=ZA
      G1(3,1)=ZA
      G1(3,2)=ZA
      G1(3,3)=ZZ
      G2(1,1)=ZZ
      G2(1,2)=ZA
      G2(1,3)=ZA
      G2(2,1)=ZA
      G2(2,2)=CP
      G2(2,3)=-SP
      G2(3,1)=ZA
      G2(3,2)=SP
      G2(3,3)=CP
      CALL INVERT (R1,3,D,LL,MM)
      DO 5 J=1,3
      DO 5 K=1,3
      R3(J,K)=ZA
```

```
      DO 5 L=1,3
    5 R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
      DO 6 J=1,3
      DO 6 K=1,3
      R1(J,K)=ZA
      DO 6 L=1,3
    6 R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
      DO 7 J=1,3
      DO 7 K=1,3
      R(J,K)=ZA
      DO 7 L=1,3
    7 R(J,K)=R(J,K)+R1(J,L)*R3(L,K)
      DO 8 J=1,6
      DO 8 K=1,6
    8 U(J,K)=ZA
      U(1,1)=ZZ
      U(2,2)=ZZ
      U(3,3)=ZZ
      U(4,4)=ZZ
      U(5,5)=ZZ
      U(6,6)=ZZ
      DO 9 L=1,3
      U(4,L)=R(1,L)
      U(5,L)=R(2,L)
    9 U(6,L)=R(3,L)
      RETURN
      END

      SUBROUTINE BRANCO(R3,VV,PHI,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R3(3,3),U(6,6),VV(6,3),R1(25,25),R2(3,3),G1(3,3),G2(3,3)
     1,S(3,3),LL(25),MM(25)
      ZZ=1.0D0
      ZA=0.0D0
      DO 4 K=1,3
      R1(1,K)=VV(1,K)
      R1(2,K)=VV(3,K)
      R1(3,K)=VV(4,K)
      R2(1,K)=VV(2,K)
      R2(2,K)=VV(5,K)
    4 R2(3,K)=VV(6,K)
      SP=DSIN(PHI)
      CP=DCOS(PHI)
      G1(1,1)=CP
      G1(1,2)=ZA
      G1(1,3)=-SP
      G1(2,1)=ZA
```

```
      G1(2,2)=ZZ
      G1(2,3)=ZA
      G1(3,1)=SP
      G1(3,2)=ZA
      G1(3,3)=CP
      G2(1,1)=CP
      G2(1,2)=SP
      G2(1,3)=ZA
      G2(2,1)=-SP
      G2(2,2)=CP
      G2(2,3)=ZA
      G2(3,1)=ZA
      G2(3,2)=ZA
      G2(3,3)=ZZ
      CALL INVERT (R1,3,D,LL,MM)
      DO 5 J=1,3
      DO 5 K=1,3
      R3(J,K)=ZA
      DO 5 L=1,3
    5 R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
      DO 6 J=1,3
      DO 6 K=1,3
      R1(J,K)=ZA
      DO 6 L=1,3
    6 R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
      DO 7 J=1,3
      DO 7 K=1,3
      S(J,K)=ZA
      DO 7 L=1,3
    7 S(J,K)=S(J,K)+R1(J,L)*R3(L,K)
      DO 8 J=1,6
      DO 8 K=1,6
    8 U(J,K)=ZA
      U(1,1)=ZZ
      U(2,1)=S(1,1)
      U(2,2)=ZZ
      U(2,3)=S(1,2)
      U(2,4)=S(1,3)
      U(3,3)=ZZ
      U(4,4)=ZZ
      U(5,1)=S(2,1)
      U(5,3)=S(2,2)
      U(5,4)=S(2,3)
      U(5,5)=ZZ
      U(6,1)=S(3,1)
      U(6,3)=S(3,2)
      U(6,4)=S(3,3)
      U(6,6)=ZZ
```

```
      RETURN
      END

      SUBROUTINE STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION SVI(6,1),UU(6,3)
      ZZ=1.0D0
      ZA=0.0D0
      M=0
      DO 1 J=1,6
      DO 1 K=1,3
    1 UU(J,K)=ZA
      IF(PM) 40,40,41
   40 DO 42 J=1,6
      IF(SVI(J,1)) 42,42,43
   43 M=M+1
      UU(J,M)=ZZ
      IF(M-1) 42,44,42
   44 UU(J,2)=-D1
      UU(J,3)=-D2
   42 CONTINUE
      RETURN
   41 P1=P1+D1
      P2=P2+D2
      IF(SS)14,15,15
   15 IF(SS-ZZ) 12,11,10
   14 IF(PP-ZZ) 12,11,10
   10 DO 4 J=1,6
      IF(SVI(J,1)) 4,4,2
    2 M=M+1
      UU(J,M)=ZZ
      IF(M-2) 4,3,5
    3 UU(J,1)=P1
      GO TO 4
    5 UU(J,1)=P2
    4 CONTINUE
      RETURN
   11 DO 21 J=1,6
      IF(SVI(J,1))21,21,22
   22 M=M+1
      UU(J,M)=ZZ
      IF(M-1) 25,26,25
   26 UU(J,3)=P1
   25 IF(M-2) 21,23,21
   23 UU(J,3)=P2
   21 CONTINUE
      RETURN
```

```
12 DO 31 J=1,6
   IF(SVI(J,1)) 31,31,32
32 M=M+1
   UU(J,M)=ZZ
   IF(M-1) 35,36,35
36 UU(J,2)=P2
35 IF(M-2) 31,31,33
33 UU(J,2)=P1
31 CONTINUE
   RETURN
   END


   SUBROUTINE STATEB(SVB,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
   IMPLICIT REAL*8(A-H,O-Z)
   DIMENSION VV(6,3),YY(3,3,25),V(3,3),SVB(6,22)
   ZZ=1.0D0
   ZA=0.0D0
   B1=ZA
   B2=ZA
   B3=ZA
   D1=ZA
   D2=ZA
   IF(PM) 33,40,22
22 IF(BRC)21,20,21
21 IF(L-1) 23,20,23
23 DO 10 J=1,3
   DO 10 K=1,3
10 V(J,K)=YY(J,K,N)
   IF(SS) 11,12,12
11 IF(PP-ZZ) 32,31,30
12 IF(SS-ZZ) 33,31,30
30 B1=V(1,1)+V(1,2)*D1+V(1,3)*D2+B1
   B2=V(2,1)+V(2,2)*D1+V(2,3)*D2+B2
   B3=V(3,1)+V(3,2)*D1+V(3,3)*D2+B3
   GO TO 33
31 B1=B1+V(1,1)*D2+V(1,3)*D1+V(1,2)
   B2=B2+V(2,1)*D2+V(2,3)*D1+V(2,2)
   B3=B3+V(3,1)*D2+V(3,3)*D1+V(3,2)
   GO TO 33
32 B1=B1+V(1,1)*D1+V(1,2)*D2+V(1,3)
   B2=B2+V(2,1)*D1+V(2,2)*D2+V(2,3)
   B3=B3+V(3,1)*D1+V(3,2)*D2+V(3,3)
33 Q1=Q1*B1
   Q2=B1*Q2+B2
   Q3=B1*Q3+B3
20 M=1
   DO 1 J=1,6
```

```
      DO 1 K=1,3
    1 VV(J,K)=ZA
      DO 4 J=1,6
      IF(SVB(J,N)) 4,4,2
    2 IF(M-1)7,3,7
    3 IF(PM) 34,35,35
   35 VV(J,1)=Q1
      GO TO 36
   34 VV(J,1)=ZZ
   36 M=M+1
      GO TO 4
    7 IF(M-2) 4,8,9
    8 VV(J,1)=Q2
      VV(J,M)=ZZ
      M=M+1
      GO TO 4
    9 VV(J,1)=Q3
      VV(J,M)=ZZ
    4 CONTINUE
      RETURN
   40 IF(FINK) 54,51,54
   54 IF(BRC) 51,51,52
   52 DO 53 J=1,3
      DO 53 K=1,3
   53 V(J,K)=YY(J,K,N)
      D1=(V(1,2)/V(1,1)+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.0D0
      D2=(V(1,3)/V(1,1)+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.0D0
   51 M=0
      DO 41 J=1,6
      DO 41 K=1,3
   41 VV(J,K)=ZA
      DO 42 J=1,6
      IF(SVB(J,N)) 42,42,43
   43 M=M+1
      VV(J,M)=ZZ
      IF(M-1) 42,44,42
   44 VV(J,2)=-D1
      VV(J,3)=-D2
   42 CONTINUE
      RETURN
      END

      SUBROUTINE BRCOR(R3,UU,XX,JK,VV,PM)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R3(3,3),UU(6,3),XX(3,3),UUU(3,3),VV(6,3)
      ZZ=1.0D0
      ZA=0.0D0
```

```
      IF(PM) 20,21,20
   20 IF(JK) 10,10,11
   10 DO 12 J=1,3
      DO 12 K=1,3
   12 UUU(J,K)=UU(J,K)
      GO TO 3
   11 DO 13 K=1,3
      UUU(1,K)=UU(1,K)
      UUU(2,K)=UU(3,K)
   13 UUU(3,K)=UU(4,K)
    3 DO 1 J=1,3
      DO 1 K=1,3
      XX(J,K)=ZA
      DO 1 L=1,3
    1 XX(J,K)=XX(J,K)+R3(J,L)*UUU(L,K)
      RETURN
   21 IF(JK) 25,25,22
   25 DO 23 J=1,3
      DO 23 K=1,3
   23 XX(J,K)=VV(J,K)
      RETURN
   22 DO 24 K=1,3
      XX(1,K)=VV(1,K)
      XX(2,K)=VV(3,K)
   24 XX(3,K)=VV(4,K)
      RETURN
      END


      SUBROUTINE DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,
     1PM,D1,D2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION UU(6,3),V(6,3),SVE(6,1),D(3,3)
      ZZ=1.0D0
      ZA=0.0D0
      L=0
      X1=ZA
      X2=ZA
      Y2=ZA
      Y1=ZA
      D1=ZA
      D2=ZA
      DO 111 J=1,3
      DO 111 K=1,3
  111 D(J,K)=ZA
      DO 91 N=1,6
      IF(SVE(N,1)) 91,2,91
    2 L=L+1
```

```
      DO 92 K=1,3
   92 V(L,K)=UU(N,K)
   91 CONTINUE
      IF(PM) 66,64,61
   64 IF(FINK) 63,62,63
   62 D1=(V(1,2)/V(1,1)+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.0D0
      D2=(V(1,3)/V(1,1)+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.0D0
      FINK=ZZ
      RETURN
   63 FINK=ZA
      RETURN
   66 RETURN
   61 IF(SS) 311,312,312
  311 IF(PP-ZZ) 400,401,402
  312 IF(SS-ZZ) 313,314,315
  402 IF(RR-ZZ) 300,403,300
  315 IF(RR-ZZ) 340,341,340
  300 IF(V(3,3)) 93,94,93
  340 IF(V(3,2)) 93,94,93
   93 DO 95 J=2,3
      DO 95 K=1,3
   95 D(J,K)=V(J,K)
      DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
      IF(DV) 98,94,98
   94 RR=ZZ
      HH=ZZ
      RETURN
  403 IF(V(2,3)) 96,200,96
  341 IF(V(2,2)) 96,200,96
   96 DO 97 K=1,3
      D(2,K)=V(3,K)
   97 D(3,K)=V(2,K)
      DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
      IF(DV) 98,200,98
   98 X1=X1-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV
      X2=X2-(V(1,2)*D(2,1)-V(1,1)*D(2,2))/DV
      IF(X1) 100,102,100
  102 IF(X2) 100,101,100
  101 FF=ZZ
      RETURN
  100 IF(SS) 354,355,355
  354 Y2=Y2-D(3,1)/D(3,3)-X1*D(3,2)/D(3,3)
      RETURN
  355 Y1=Y1-D(3,1)/D(3,2)-X2*D(3,3)/D(3,2)
      RETURN
   99 IF(SS) 20,21,21
   20 PP=ZZ
      GO TO 23
```

```
21  SS=ZZ
23  RR=ZA
    HH=ZZ
    RETURN
314 IF(RR-ZZ) 330,331,330
401 IF(RR-ZZ) 301,404,301
330 IF(V(3,1)) 81,82,81
301 IF(V(3,2)) 81,82,81
81  DO 83 J=2,3
    DO 83 K=1,3
83  D(J,K)=V(J,K)
    DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)
    IF(DV) 86,82,86
82  RR=ZZ
    HH=ZZ
    RETURN
331 IF(V(2,1)) 84,70,84
404 IF(V(2,2)) 84,70,84
84  DO 85 K=1,3
    D(2,K)=V(3,K)
85  D(3,K)=V(2,K)
    DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)
    IF(DV) 86,70,86
86  X1=X1-(V(1,3)*D(2,2)-V(1,2)*D(2,3))/DV
    X2=X2-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV
    IF(X1) 203,201,203
201 IF(X2) 203,202,203
202 FF=ZZ
    RETURN
203 IF(SS) 352,353,353
353 Y1=Y1-D(3,3)/D(3,1)-X2*D(3,2)/D(3,1)
    RETURN
352 Y2=Y2-D(3,3)/D(3,2)-X1*D(3,1)/D(3,2)
    RETURN
70  IF(SS) 25,24,24
24  SS=2.0D0
    GO TO 26
25  PP=2.0D0
26  RR=ZA
    HH=ZZ
    RETURN
313 IF(RR-ZZ) 320,321,320
400 IF(RR-ZZ) 302,405,302
320 IF(V(3,3)) 71,72,71
302 IF(V(3,1)) 71,72,71
71  DO 73 J=2,3
    DO 73 K=1,3
73  D(J,K)=V(J,K)
```

```
      DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
      IF(DV) 76,72,76
  72  RR=ZZ
      HH=ZZ
      RETURN
 405  IF(V(2,1)) 74,99,74
 321  IF(V(2,3)) 74,99,74
 200  KKK=1
      RR=ZA
      RETURN
  74  DO 75 K=1,3
      D(2,K)=V(3,K)
  75  D(3,K)=V(2,K)
      DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
      IF(DV) 76,99,76
  76  X2=X2-(V(1,2)*D(2,3)-V(1,3)*D(2,2))/DV
      X1=X1-(V(1,1)*D(2,2)-D(2,1)*V(1,2))/DV
      IF(X1) 213,211,213
 211  IF(X2) 213,212,213
 212  FF=ZZ
      RETURN
 213  IF(SS) 350,351,351
 351  Y1=Y1-(D(3,2)+X2*D(3,1))/D(3,3)
      RETURN
 350  Y2=Y2-(D(3,2)+X1*D(3,3))/D(3,1)
      RETURN
      END
```

BIBLIOGRAPHY

1. Kim, Su Y., "Vibration Analysis of Piping Systems," Master's Degree Thesis, U.S. Naval Postgraduate School, 1966.

2. Fink, George E., "Vibration Analysis of Piping Systems," Master's Degree Thesis, U.S. Naval Postgraduate School, 1964.

3. Teledyne Materials Research Company Report E-1086, Piping Design Guide, June 27, 1969.

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1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

VIBRATION OF AN ELL-SHAPED UNIFORM FILAMENT WITH CLAMPED ENDS

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; December 1970

5. AUTHOR(S) (First name, middle initial, last name)

Cetin Derin, Lieutenant (junior grade), Turkish Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1970	116	3

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotatory inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.

Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
VIBRATION						
PIPE VIBRATION						
VIBRATION OF ELL-SHAPED BAR						

Thesis 124421
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c.1 Vibration of an
ell-shaped uniform
filament with clamped
ends.